# Quantum Mechanics of Identity

## Bosons, Fermions, Anyons

- Concept of quantum statistics
- Why there are only bosons and fermions (D = 3)
- Two deep issues around bosons and fermions
- Braid group and anyons (D = 2)
- Gauge theory of anyons
- Anyons in nature
- Engineering anyons

# Concept of Quantum Statistics

In classical physics, nothing much hinges on whether particles are slightly different versus strictly indistinguishable.

Thus, classical physics leaves one of the most striking features of Nature looking gratuitous - indeed, puzzling.

In quantum physics it is different.  There are phenomena which can occur only for particles which are strictly indistinguishable, and which are observed to occur.  We call these phenomena “quantum statistics”.

Quantum statistics is profoundly important, both conceptually and practically.

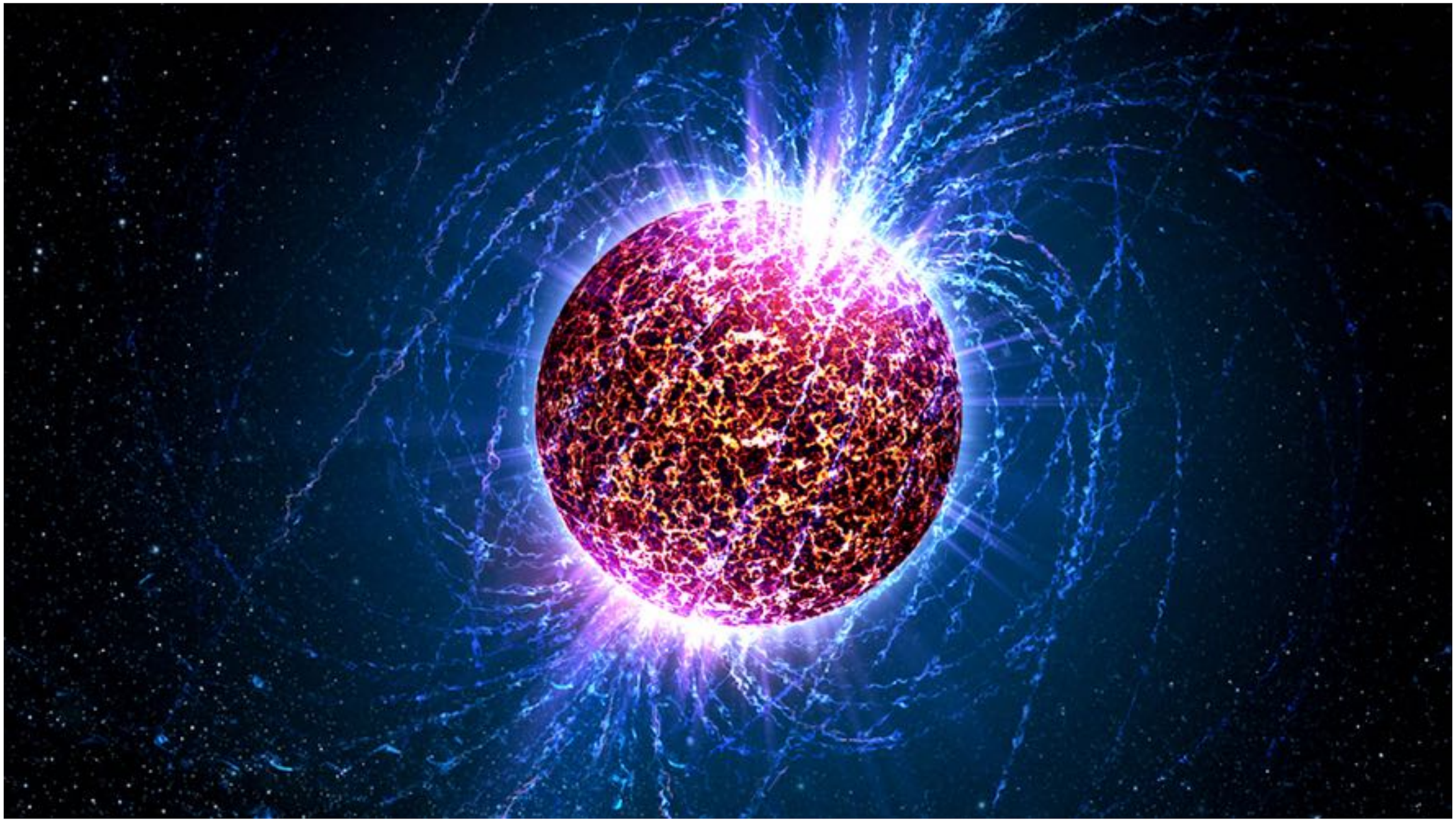
In conventional textbooks, quantum statistics is presented as a three-part canon law:

1. Members of observed classes of particles, e.g. electrons, photons, *nuclei*, *atoms,* … are strictly indistinguishable
   (if they are in the same internal state).

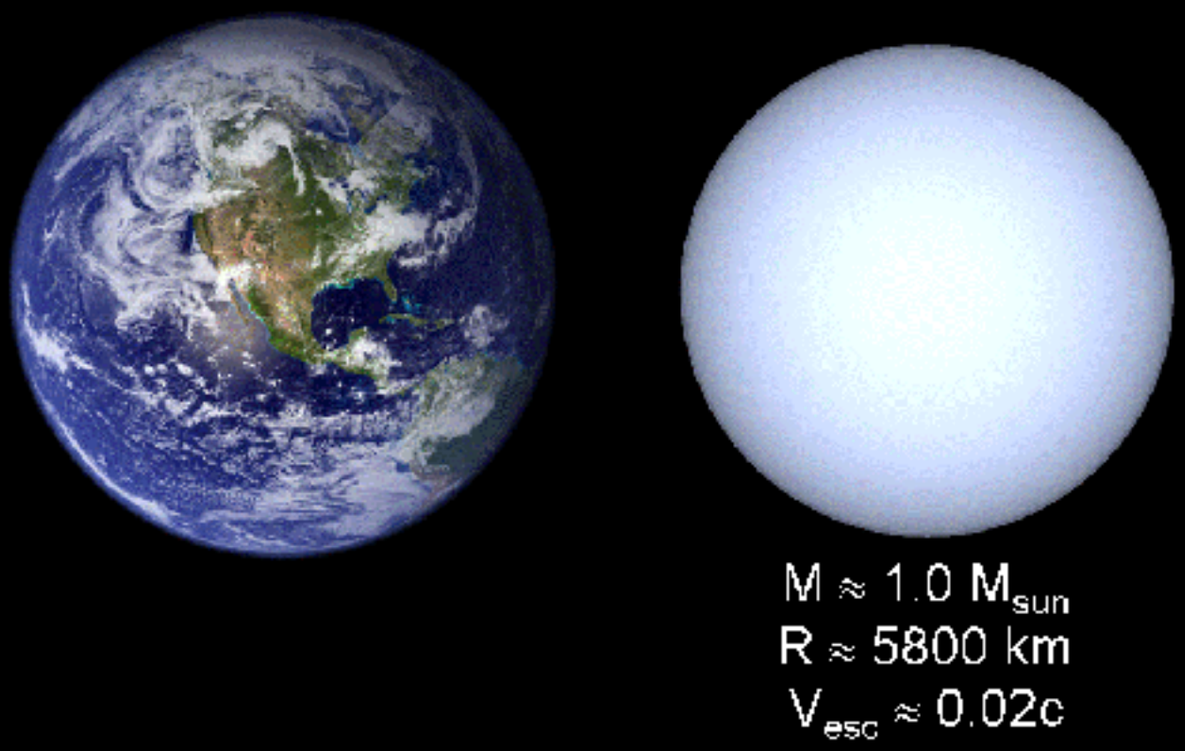
2. There are two kingdoms, the kingdom of bosons and the kingdom of fermions.

3. For bosons, you use symmetric wave functions; for fermions, you use antisymmetric wave functions.

Given those rules, and the other laws of quantum physics, you can derive marvelous consequences:



**Neutron star: gravity balanced by quantum statistics**




**White dwarf: gravity balanced by quantum statistics (electrons)**

**A giant atom**







**Neutron star: gravity balanced by quantum statistics (neutrons)**

**A giant nucleus**

| Group → / Period ↓ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 H | | | | | | | | | | | | | | | | | 2 He |
| 2 | 3 Li | 4 Be | | | | | | | | | | | 5 B | 6 C | 7 N | 8 O | 9 F | 10 Ne |
| 3 | 11 Na | 12 Mg | | | | | | | | | | | 13 Al | 14 Si | 15 P | 16 S | 17 Cl | 18 Ar |
| 4 | 19 K | 20 Ca | 21 Sc | 22 Ti | 23 V | 24 Cr | 25 Mn | 26 Fe | 27 Co | 28 Ni | 29 Cu | 30 Zn | 31 Ga | 32 Ge | 33 As | 34 Se | 35 Br | 36 Kr |
| 5 | 37 Rb | 38 Sr | 39 Y | 40 Zr | 41 Nb | 42 Mo | 43 Tc | 44 Ru | 45 Rh | 46 Pd | 47 Ag | 48 Cd | 49 In | 50 Sn | 51 Sb | 52 Te | 53 I | 54 Xe |
| 6 | 55 Cs | 56 Ba | 57 La | * 72 Hf | 73 Ta | 74 W | 75 Re | 76 Os | 77 Ir | 78 Pt | 79 Au | 80 Hg | 81 Tl | 82 Pb | 83 Bi | 84 Po | 85 At | 86 Rn |
| 7 | 87 Fr | 88 Ra | 89 Ac | * 104 Rf | 105 Db | 106 Sg | 107 Bh | 108 Hs | 109 Mt | 110 Ds | 111 Rg | 112 Cn | 113 Nh | 114 Fl | 115 Mc | 116 Lv | 117 Ts | 118 Og |
| | | | | * 58 Ce | 59 Pr | 60 Nd | 61 Pm | 62 Sm | 63 Eu | 64 Gd | 65 Tb | 66 Dy | 67 Ho | 68 Er | 69 Tm | 70 Yb | 71 Lu | |
| | | | | * 90 Th | 91 Pa | 92 U | 93 Np | 94 Pu | 95 Am | 96 Cm | 97 Bk | 98 Cf | 99 Es | 100 Fm | 101 Md | 102 No | 103 Lr | |



But two obvious questions arise:

Why are there two kingdoms, instead of one?

Why are there two kingdoms, instead of many?

In recent years, those questions have been very fruitful.

# Why There Are Only Bosons and Fermions (D = 3)

Quantum mechanics is a system of assigning amplitudes to possible histories.  The rules must be consistent with certain general principles:

1. The total amplitude is the sum of amplitudes over all ways to get from the start to the finish.

2. The amplitudes for successive histories multiply.

3. The amplitudes for energetic histories should correspond to classical physics.

4. The amplitudes for nearby histories should vary continuously.

Let us consider applying this framework to two indistinguishable particles.

Rules 3 and 4 almost dictate the result - but not quite.

If our particles start at A, B and end at A, B they might or might not have exchanged places.

Those are *topologically* distinct possibilities, so rules 3 and 4 don’t tell us about their *relative* weight.

But if we repeat the process, we get two histories which *can* be continuously connected.

[belt demonstration]

In that case, there is no non-trivial relative weight.

Thus, according to rule 2, the relative weight w must satisfy $w^2 = 1$.

w = 1 is bosons, w = -1 is fermions.

# Two Deep Issues

The quantum statistics of a particle is related to its spin.

Particles whose spin is half an odd integer (in units of Planck’s constant) are fermions, particles whose spin is an integer are bosons.

The spin of a particle tells you how its wave-function changes as you rotate it.

For particles with half-odd integer spin, the wave function changes sign after rotation through 2π.

That strange fact is logically connected to the strange fact of fermion statistics, as this “ribbon argument” displays:

[ribbon demonstration]

Ordinary symmetries do not relate particles of different spin.

*Supersymmetry* connects the two kingdoms.

There is (very) circumstantial evidence that supersymmetry is a feature of fundamental physical law, but nothing definitive.

# Braid Group and Anyons (D = 2)



**Jon Leinaas**



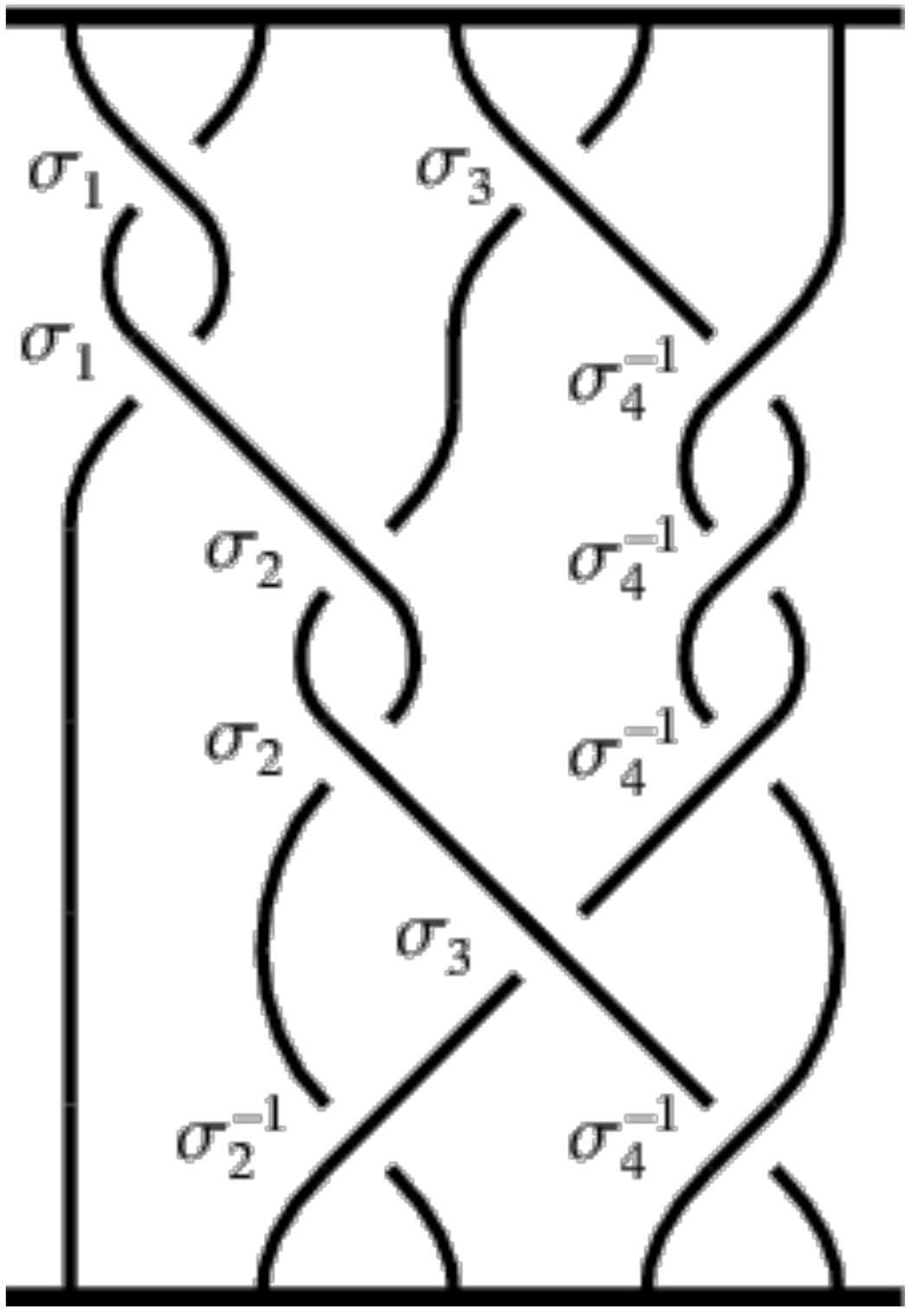
**Jan Myrheim**

The belt argument required an “excursion” into the third dimension.

In two space dimensions, you manifestly can’t undo a double exchange - one guy has circled around the other!

The topology of paths in 2 dimensions is described by the *braid group*.


www.barefootblonde.com


$\sigma_1$
$\sigma_3$
$\sigma_1$
$\sigma_4^{-1}$
$\sigma_2$
$\sigma_4^{-1}$
$\sigma_2$
$\sigma_4^{-1}$
$\sigma_3$
$\sigma_2^{-1}$
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The simplest anyons simply assign a phase proportional to the total winding.

This manifestly obeys rule 2, whatever the proportionality constant.

Bosons and fermions are special cases.

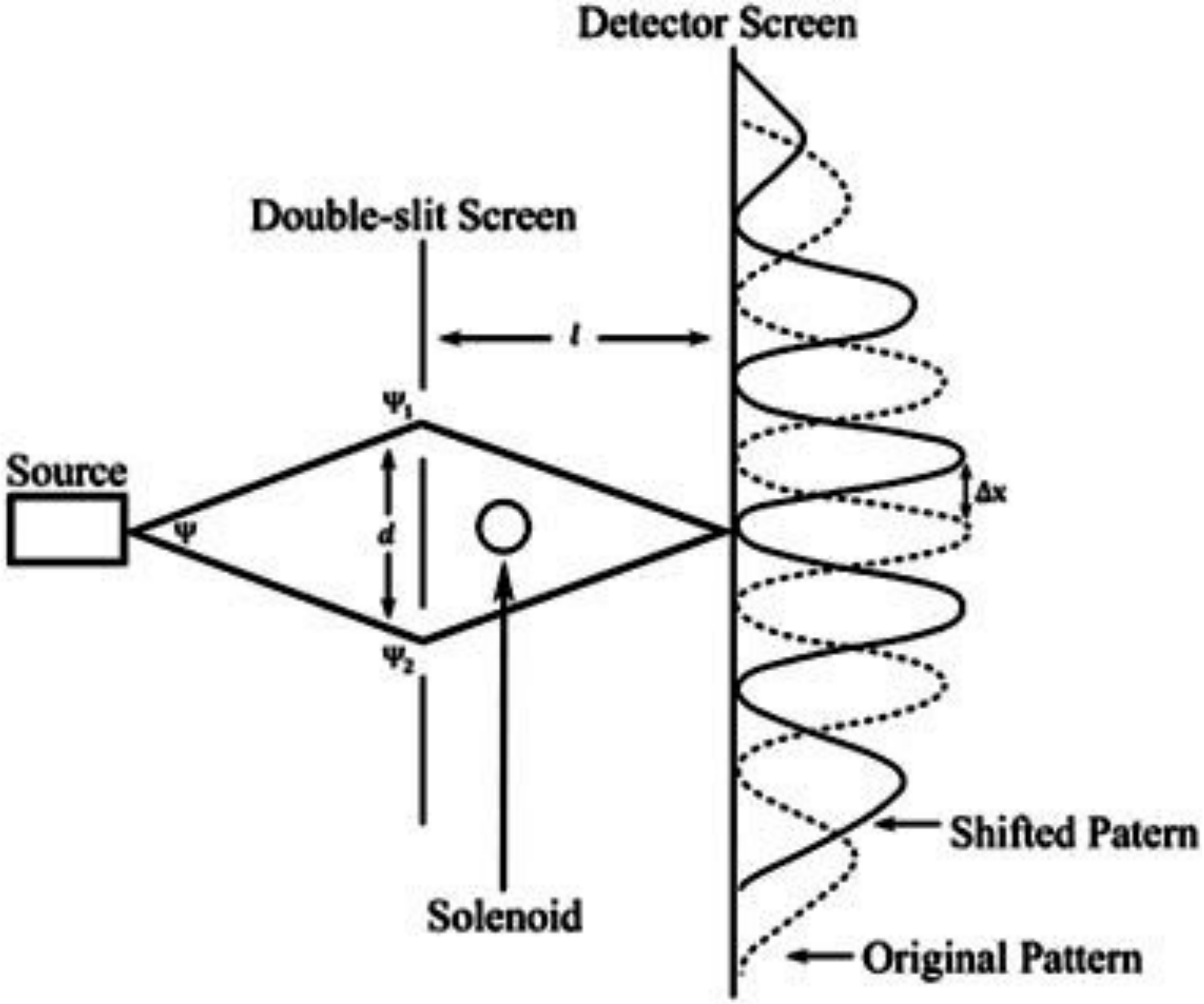
One can also represent the braid group using non-commuting matrices.  That defines “non-abelian” anyons.

This is a more sophisticated implementation of rule 2.

Since there can be winding even without exchange, one can also have “mutual statistics” between *distinguishable* particles,

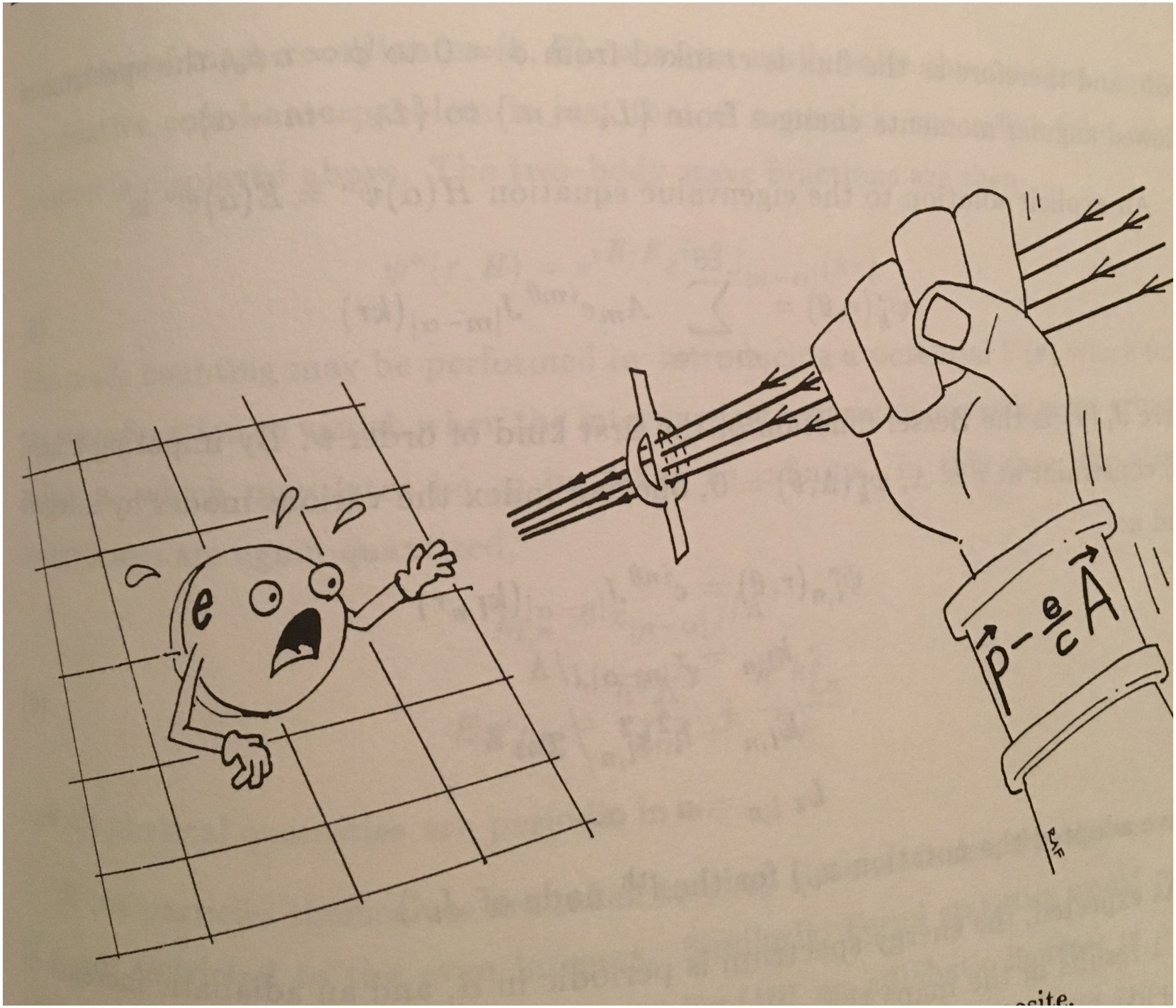
# Gauge Theory of Anyons

The wave function of charged particle encircling a magnetic flux tube (solenoid) acquires a phase, even though it feels no force.  This is known as the Aharonov-Bohm effect.


Detector Screen
Double-slit Screen
l
Ψ1
Source
Ψ
d
Ψ2
Δx
Shifted Patern
Solenoid
Original Pattern

Note that in 2 dimensions a magnetic flux tube or solenoid reduces to a point, so it can be associated to a particle.

Thus we can model anyons, or change the quantum statistics of particles, by associating emergent (“fictitious”) charge and flux to particles.

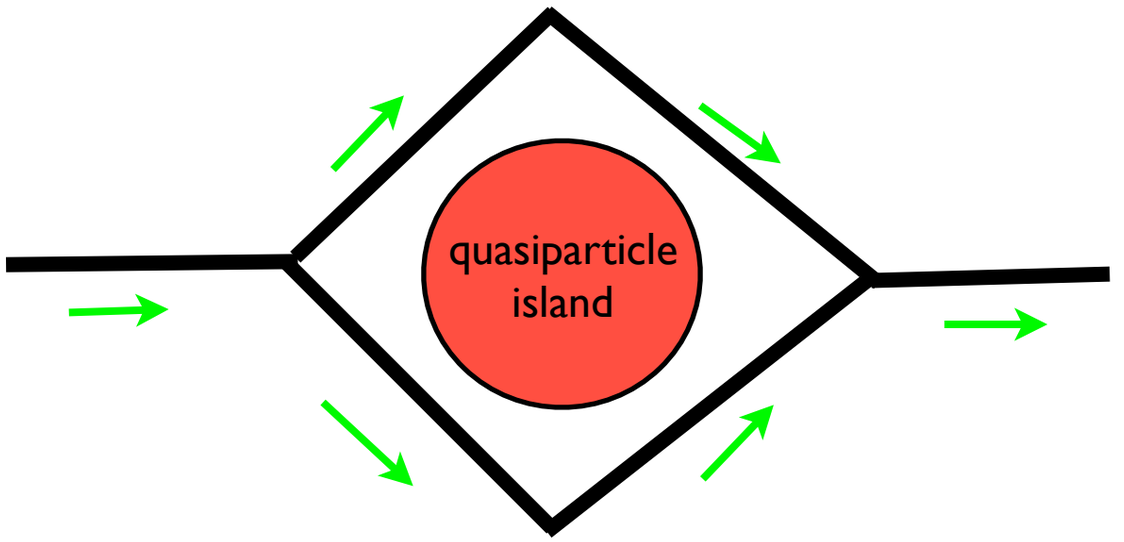

e
$\vec{P} - \frac{e}{c}\vec{A}$
RAF

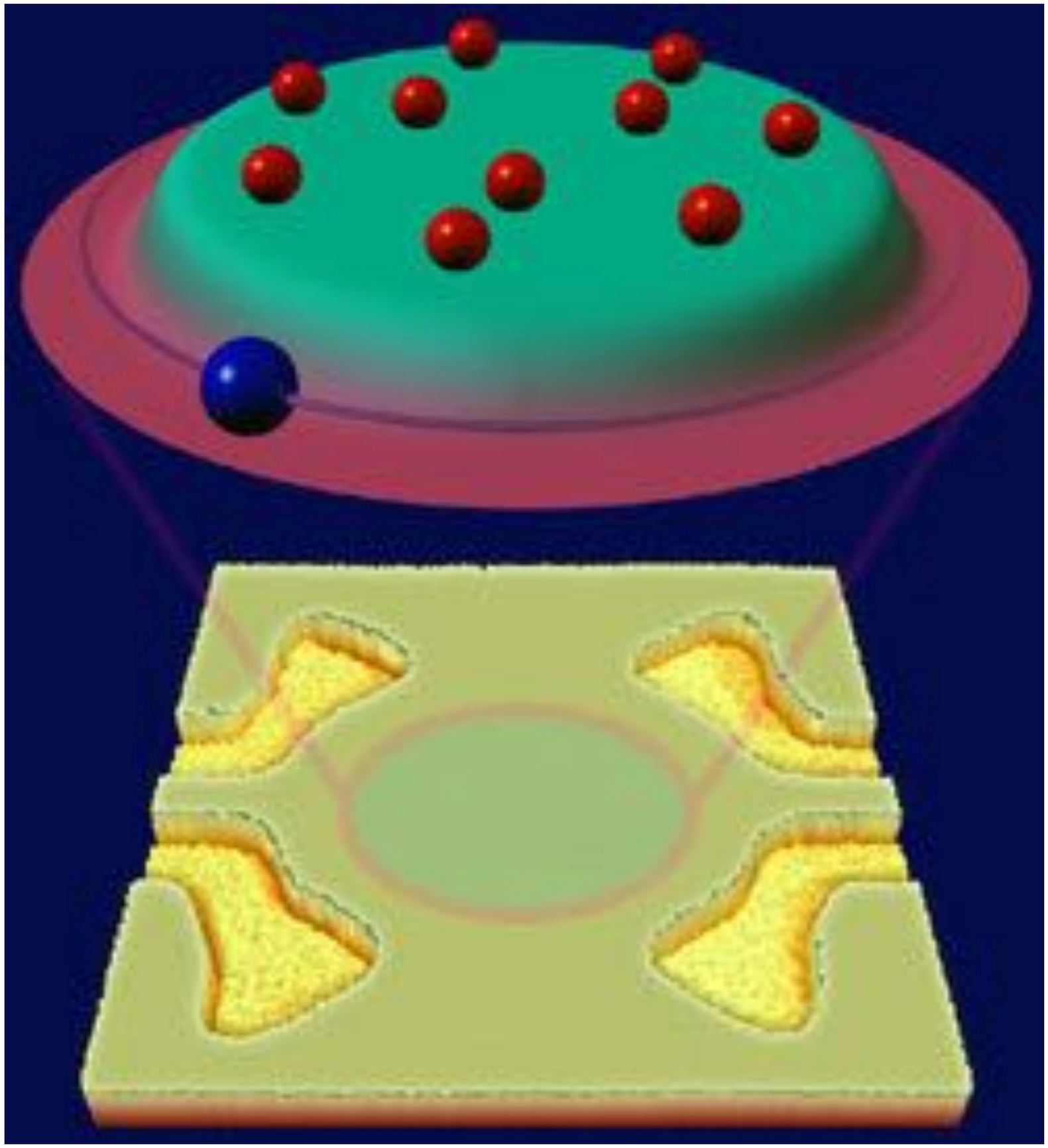
This charge-flux construction generalizes in natural way to include non-abelian and mutual statistics.

# Anyons in Nature

The existence of anyons - of many kinds - is firmly established theoretically, and by numerical simulation, in the fractional quantum Hall effect.

But direct experimental demonstration, for example through interferometry, remains an open challenge.


quasiparticle
island



Anyons are also predicted to exist in other states of matter, such as Chern insulators and spin liquids, which might offer cleaner access.

# Engineering Anyons

Quantum simulation of anyon behavior has been achieved.

# Emulating anyonic fractional statistical behavior in a superconducting quantum circuit

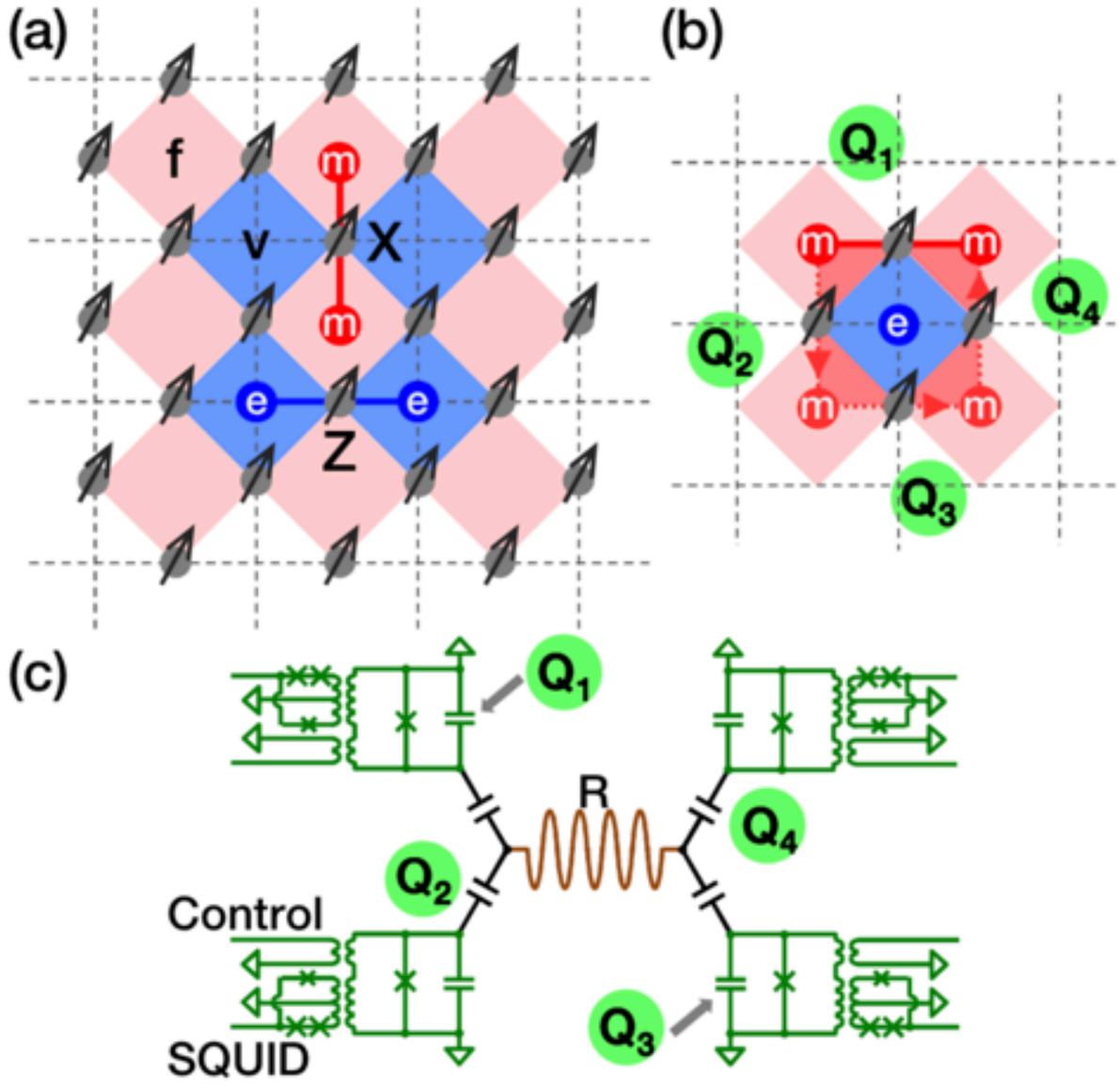
Y. P. Zhong[1,2], D. Xu[1], P. Wang[1], C. Song[1], Q. J. Guo[1], W. X. Liu[1], K. Xu[1], B. X. Xia[2], C.-Y. Lu[2,3],* Siyuan Han[4], Jian-Wei Pan[2,3],† and H. Wang[1,2]‡

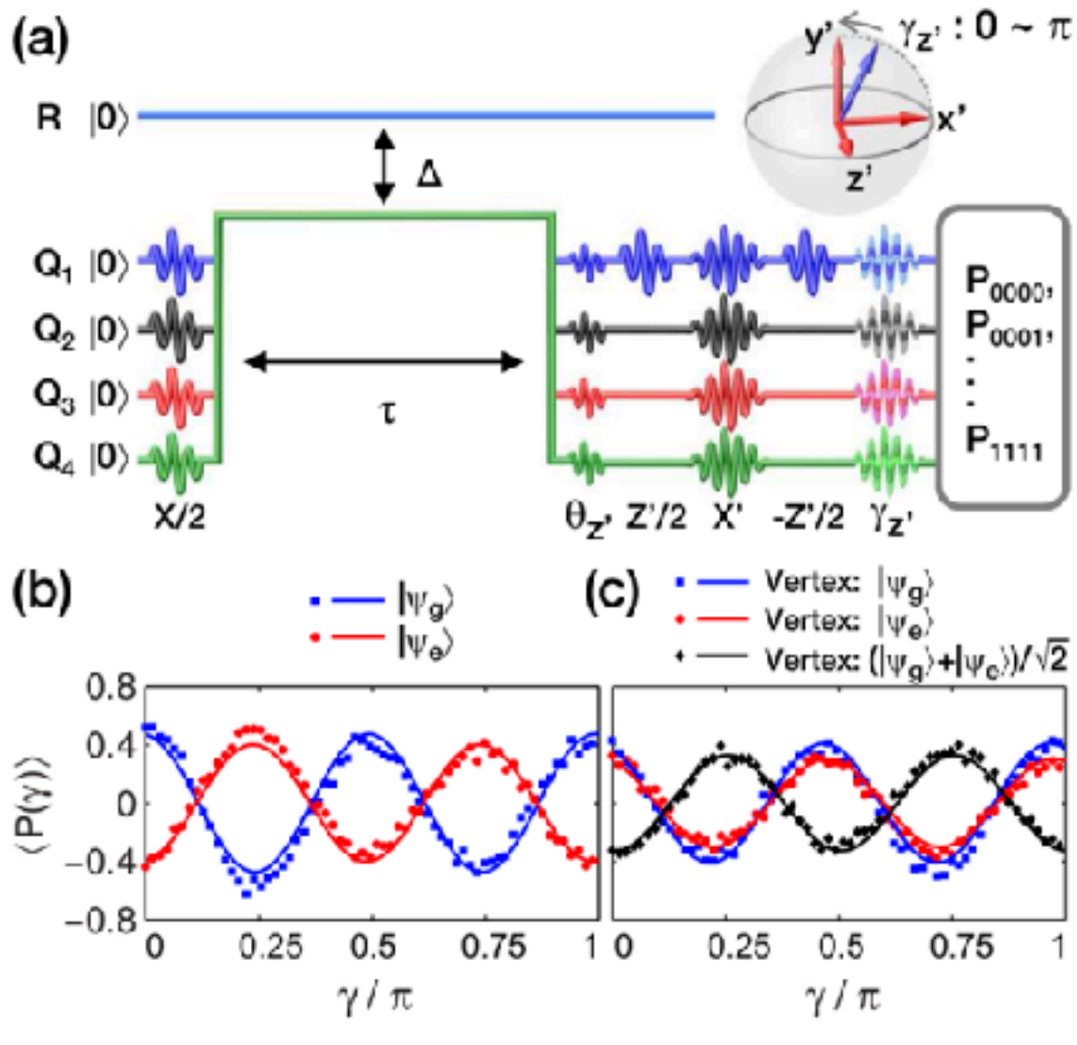
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(Dated: September 11, 2018)

Anyons are exotic quasiparticles obeying fractional statistics, whose behavior can be emulated in artificially designed spin systems. Here we present an experimental emulation of creating anyonic excitations in a superconducting circuit that consists of four qubits, achieved by dynamically generating the ground and excited states of the toric code model, i.e., four-qubit Greenberger-Horne-Zeilinger states. The anyonic braiding is implemented via single-qubit rotations: a phase shift of $\pi$ related to braiding, the hallmark of Abelian 1/2 anyons, has been observed through a Ramsey-type interference measurement.

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(a)
f
v
X
Z
m
e
(b)
Q1
Q2
Q3
Q4
(c)
Control
SQUID
R

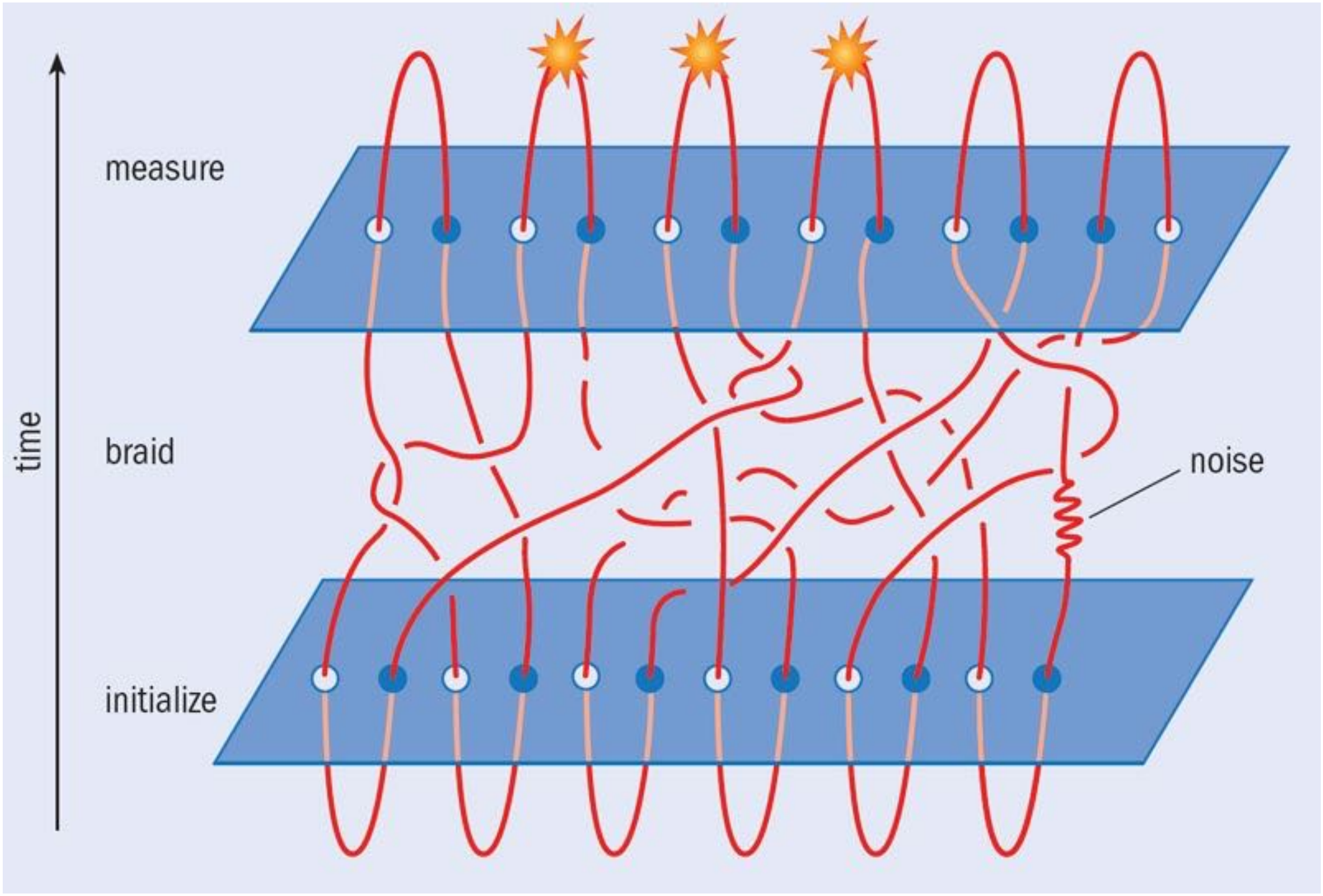

(a)
R |0⟩
Δ
Q1 |0⟩
Q2 |0⟩
Q3 |0⟩
Q4 |0⟩
τ
X/2
θZ' Z'/2 X' -Z'/2 γZ'
P0000
P0001
P1111
(b)
|ψg⟩
|ψe⟩
⟨P(γ)⟩
γ / π
(c)
Vertex: |ψg⟩
Vertex: |ψe⟩
Vertex: (|ψg⟩+|ψe⟩)/√2
γ / π

An enormous effort, lavishly funded by Microsoft, is being directed at producing and controlling “Majorana” anyons (majorinos).

Majorinos are non-abelian anyons associated with specially designed vortex-like structures in superconductors.

They could provide building blocks for topological quantum computing.


Epitaxial Majorana Device
a
Ti/Au
Ti/Al/V
InAs
Al
A
500 nm


measure
braid
initialize
noise
time

At the moment, the experimenters have produced individual Majorinos, but they have not demonstrated their statistics directly.

That demonstration will be an early step in Microsoft's ambitious program to make and manipulate many Majorinos within a scalable architecture.

It will be disappointing - and surprising - if it is not accomplished soon.