All too often, we ignore goals, genres, or values, or we assume that they are so apparent that we do not bother to highlight them. Yet judgments about whether an exercise – a paper, a project, an essay response on an examination – has been done intelligently or stupidly are often difficult for students to fathom. And since these evaluations are not well understood, few if any lessons can be drawn from them. Laying out the criteria by which judgments of quality are made may not suffice in itself to improve quality, but in the absence of such clarification, we have little reason to expect our students to go about their work intelligently.

Frank Wilczek

on the world's numerical recipe

Twentieth-century physics began around 600 B.C. when Pythagoras of Samos proclaimed an awesome vision.

By studying the notes sounded by plucked strings, Pythagoras discovered that the human perception of harmony is connected to numerical ratios. He examined strings made of the same material, having the same thickness, and under the same tension, but of different lengths. Under these conditions, he found that the notes sound harmonious precisely when the ratio of the lengths of string can be expressed in small whole numbers. For example, the length ratio

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Frank Wilczek, Herman Feshbach Professor of Physics at MIT, is known, among other things, for the discovery of asymptotic freedom, the development of quantum chromodynamics, the invention of axions, and the discovery and exploitation of new forms of quantum statistics (anyons). When only twenty-one years old and a graduate student at Princeton University, he and David Gross defined the properties of gluons, which hold atomic nuclei together. He has been a Fellow of the American Academy since 1993.

2:1 sounds a musical octave, 3:2 a musical fifth, and 4:3 a musical fourth.

The vision inspired by this discovery is summed up in the maxim "All Things are Number." This became the credo of the Pythagorean Brotherhood, a mixed-sex society that combined elements of an archaic religious cult and a modern scientific academy.

The Brotherhood was responsible for many fine discoveries, all of which it attributed to Pythagoras. Perhaps the most celebrated and profound is the Pythagorean Theorem. This theorem remains a staple of introductory geometry courses. It is also the point of departure for the Riemann-Einstein theories of curved space and gravity.

Unfortunately, this very theorem undermined the Brotherhood's credo. Using the Pythagorean Theorem, it is not hard to prove that the ratio of the hypotenuse of an isosceles right triangle to either of its two shorter sides cannot be expressed in whole numbers. A member of the Brotherhood who revealed this dreadful secret drowned shortly afterwards, in suspicious circumstances. Today, when we say $\sqrt{2}$ is irrational, our language still reflects these ancient anxieties.

Still, the Pythagorean vision, broadly understood – and stripped of cultic, if not entirely of mystical, trappings – remained for centuries a touchstone for pioneers of mathematical science. Those working within this tradition did not insist on whole numbers, but continued to postulate that the deep structure of the physical world could be captured in purely conceptual constructions. Considerations of symmetry and abstract geometry were allowed to supplement simple numerics.

In the work of the German astronomer Johannes Kepler (1570–1630), this program reached a remarkable apotheosis – only to unravel completely.

The world's numerical recipe

Students today still learn about Kepler's three laws of planetary motion. But before formulating these celebrated laws, this great speculative thinker had announced another law – we can call it Kepler's zeroth law – of which we hear much less, for the very good reason that it is entirely wrong. Yet it was his discovery of the zeroth law that fired Kepler's enthusiasm for planetary astronomy, in particular for the Copernican system, and launched his extraordinary career. Kepler's zeroth law concerns the relative size of the orbits of different planets. To formulate it, we must imagine that the planets are carried about on concentric spheres around the Sun. His law states that the successive planetary spheres are of such proportions that they can be inscribed within and circumscribed about the five Platonic solids. These five remarkable solids – tetrahedron, cube, octahedron, dodecahedron, icosahedron – have faces that are congruent equilateral polygons. The Pythagoreans studied them, Plato employed them in the speculative cosmology of the *Timaeus*, and Euclid climaxed his *Elements* with the first known proof that only five such regular polyhedra exist.

Kepler was enraptured by his discovery. He imagined that the spheres emitted music as they rotated, and he even speculated on the tunes. (This is the source of the phrase "music of the spheres.") It was a beautiful realization of the Pythagorean ideal. Purely conceptual, yet sensually appealing, the zeroth law seemed a production worthy of a mathematically sophisticated Creator.

To his great credit as an honest man and – though the concept is anachronistic – as a scientist, Kepler did not wallow in mystic rapture, but actively strove to see whether his law accurately matched reality. He discovered that it does not. In wrestling with the precise observations of Tycho Brahe, Kepler was forced to give up circular in favor of elliptical orbits. He couldn't salvage the ideas that first inspired him.

After this, the Pythagorean vision went into a long, deep eclipse. In Newton's classical synthesis of motion and gravitation, there is no sense in which structure is governed by numerical or conceptual constructs. All is dynamics. Newton's laws inform us, given the positions, velocities, and masses of a system of gravitating bodies at one time, how they will move in the future. They do not fix a unique size or structure for the solar system. Indeed, recent discoveries of planetary systems around distant stars have revealed quite different patterns. The great developments of nineteenth-century physics, epitomized in Maxwell's equations of electrodynamics, brought many new phenomena with the scope of physics, but they did not alter this situation essentially. There is nothing in the equations of classical physics that can fix a definite scale of size, whether for planetary systems, atoms, or anything else. The world-system of classical physics is divided between initial conditions that can be assigned arbitrarily, and dynamical equations. In those equations, neither whole numbers nor any other purely conceptual elements play a distinguished role.

Quantum mechanics changed everything.

Emblematic of the new physics, and decisive historically, was Niels Bohr's atomic model of 1913. Though it applies in a vastly different domain, Bohr's model of the hydrogen atom bears an uncanny resemblance to Kepler's system of planetary spheres. The binding force is electrical rather than gravitational, the players are electrons orbiting around protons rather than planets orbiting the Sun, and the size is a factor 10^{-22} smaller; but the leitmotif of Bohr's model is unmistakably "Things are Number."

Through Bohr's model, Kepler's idea that the orbits that occur in nature are precisely those that embody a conceptual ideal emerged from its embers, reborn like a phoenix, after three hundred years' quiescence. If anything, Bohr's model conforms more closely to the Pythagorean ideal than Kepler's, since its preferred orbits are defined by whole numbers rather than geometric constructions. Einstein responded with great empathy and enthusiasm, referring to Bohr's work as "the highest form of musicality in the sphere of thought."

Later work by Heisenberg and Schrödinger, which defined modern quantum mechanics, superseded Bohr's model. This account of subatomic matter is less tangible than Bohr's, but ultimately much richer. In the Heisenberg-Schrödinger theory, electrons are no longer particles moving in space, elements of reality that at a given time are "just there and not anywhere else." Rather, they define oscillatory, space-filling wave patterns always "here, there, and everywhere." Electron waves are attracted to a positively charged nucleus and can form localized standing wave patterns around it. The mathematics describing the vibratory patterns that define the states of atoms in quantum mechanics is identical to that which describes the resonance of musical instruments. The stable states of atoms correspond to pure tones. I think it's fair to say that the musicality Einstein praised in Bohr's model is, if anything, heightened in its progeny (though Einstein himself, notoriously, withheld his approval from the new quantum mechanics).

The big difference between nature's instruments and those of human construction is that her designs depend not on craftsmanship refined by experience, but rather on the ruthlessly precise application of simple rules. Now if you browse through a textbook on atomic quantum mechanics, or look at atomic vibration patterns using modern visualization tools, "simple" might not be the word that leaps to mind. But it has a precise, objective meaning in this context. A theory is simpler the fewer nonconceptual elements, which must be taken from observation, enter into its construction. In this sense, Kepler's zeroth law provided a simpler (as it turns out, too simple) theory of the solar system than Newton's, because in Newton's theory the relative sizes of planetary orbits must be taken from observation, whereas in Kepler's they are determined conceptually.

From this perspective, modern atomic theory is extraordinarily simple. The Schrödinger equation, which governs electrons in atoms, contains just two nonconceptual quantities. These are the mass of the electron and the so-called fine-structure constant, denoted α, that specifies the overall strength of the electromagnetic interaction. By solving this one equation, finding the vibrations it supports, we make a concept-world that reproduces a tremendous wealth of real-world data, notably the accurately measured spectral lines of atoms that encode their inner structure. The marvelous theory of electrons and their interactions with light is called quantum electrodynamics, or QED.

In the initial modeling of atoms, the focus was on their accessible, outlying parts, the electron clouds. The nuclei of atoms, which contain most of their mass and all of their positive charge, were treated as so many tiny (but very heavy!) black boxes, buried in the core. There was no theory for the values of nuclear masses or their other properties; these were simply taken from experiment. That pragmatic approach was extremely fruitful and to this day provides the working basis for practical applications of physics in chemistry, materials science, and biology. But it failed to provide a theory that was in our sense simple, and so it left the ultimate ambitions of a Pythagorean physics unfulfilled.

The world's numerical recipe

Starting in the early 1930s, with electrons under control, the frontier of fundamental physics moved inward, to the nuclei. This is not the occasion to recount the complex history of the heroic constructions and ingenious deductions that at last, after fifty years of strenuous international effort, fully exposed the secrets of this inaccessible domain. Fortunately, the answer is easier to describe, and it advances and consummates our theme.

The theory that governs atomic nuclei is quantum chromodynamics, or QCD. As its name hints, QCD is firmly based on quantum mechanics. Its mathematical basis is a direct generalization of QED, incorporating a more intricate structure supporting enhanced symmetry. Metaphorically, QCD stands to QED as an icosahedron stands to a triangle. The basic players in QCD are quarks and gluons. For constructing an accurate model of ordinary matter just two kinds of quarks, called up and down or simply *u* and *d*, need to be considered. (There are four other kinds, at least, but they are highly unstable and not important for ordinary matter.) Protons, neutrons, π mesons, and a vast zoo of very short-lived particles called resonances are constructed from these building blocks. The particles and resonances observed in the real word match the resonant wave patterns of quarks and gluons in the concept-world of QCD, much as states of atoms match the resonant wave patterns of electrons. You can predict their masses and properties directly by solving the equations.

A peculiar feature of QCD, and a major reason why it was hard to discover, is that the quarks and gluons are never found in isolation, but always in complex associations. QCD actually predicts this "confinement" property, but that's not easy to prove.

Considering how much it accounts for, QCD is an amazingly simple theory, in our objective sense. Its equations contain just three nonconceptual ingredients: the masses of the *u* and *d* quarks and the strong coupling constant α_S, analogous to the fine structure constant of QED, which specifies how powerfully quarks couple to gluons. The gluons are automatically massless.

Actually even three is an overestimate. The quark-gluon coupling varies with distance, so we can trade it in for a unit of distance. In other words, mutant QCDs with different values of α_S generate concept-worlds that behave identically, but use different-sized metersticks. Also, the masses of the *u* and *d* quarks turn out not to be very important, quantitatively. Most of the mass of strongly interacting particles is due to the pure energy of the moving quarks and gluons they contain, according to the converse of Einstein's equation, $m = E/c^2$. The masses of the *u* and *d* quarks are much smaller than the masses of the protons and other particles that contain them.

Putting all this together, we arrive at a most remarkable conclusion. To the extent that we are willing to use the proton itself as a meterstick, and ignore the small corrections due to the *u* and *d* quark masses, QCD becomes a theory with *no nonconceptual elements whatsoever*.

Let me summarize. Starting with precisely four numerical ingredients, which must be taken from experiment, QED and QCD cook up a concept-world of mathematical objects whose behavior matches, with remarkable accuracy, the behavior of real-world matter. These objects are vibratory wave patterns. Stable elements of reality – protons, atomic nuclei, atoms – correspond, not just metaphorically but with mathematical precision, to pure tones. Kepler would be pleased.

This tale continues in several directions. Given two more ingredients, Newton's constant G_N and Fermi's constant G_F, which parametrize the strength of gravity and of the weak interaction, respectively, we can expand our concept-world beyond ordinary matter to describe virtually all of astrophysics. There is a brilliant series of ideas involving unified field theories and supersymmetry that might allow us to get by with just five ingredients. (Once you're down to so few, each further reduction marks an epoch.) These ideas will be tested decisively in coming years, especially as the Large Hadron Collider (LHC) at CERN, near Geneva, swings into operation around 2007.

On the other hand, if we attempt to do justice to the properties of many exotic, short-lived particles discovered at high-energy accelerators, things get much more complicated and unsatisfactory. We have to add pinches of many new ingredients to our recipe, until it may seem that rather than deriving a wealth of insight from a small investment of facts, we are doing just the opposite. That's the state of our knowledge of fundamental physics today – simultaneously triumphant, exciting, and a mess.

The last word I leave to Einstein:

> I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no

arbitrary constants . . . that is to say, nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory).

David G. Nathan

on clinical research & the future of medicine

Biomedical inquiry as it is practiced in America today is an amalgam of three different kinds of research: basic research, population research, and clinical research. While all three are of critical importance, it is clinical research that underpins our national medical efforts. Only clinical researchers are able to apply the knowledge of the cell and organ systems developed by basic researchers, and the population data gathered by epidemiologists and biostatisticians, to patients, making this knowledge and data relevant to medical practice by "translating" it into novel

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