QCD Meets BCS

A Colorful Story

The behavior of hadrons at high density is intrinsically interesting, and it is important for understanding nuclear matter and neutron stars.

It turns out to be amenable to a color-enhanced version of BCS theory.

The resulting theory is quite pretty, and it sheds considerable light on the classic "hard questions" of QCD, i.e.

confinement

chiral symmetry breaking

How should we approach high-density QCD?

Let us follow the Jesuit credo: "It is more blessed to ask forgiveness than permission." :

Start with quarks and gluons

Observe that high energy ⇒ apparent simplification (asymptotic freedom)

... BUT problems arise. Gapless quarks and

Fortunately, we stand on the shoulders of giants!

Superconductivity, now with color in place of electromagnetism, saves the day.

Color Superconductivity

Quark-quark forces can be attractive, even at the level of Coulomb's law!

The totally antisymmetric channel is favorable.

This triggers color superconductivity, in a very specific form.

What could it mean?

Superconductivity-QCD Dictionary

Cooper instability / Infrared divergence

Meissner effect / Color confinement (= BEH mechanism)

Gap / Chiral symmetry breaking (= Majorana mass)

Three massless flavors give a result that is quite interesting, and clean to interpret.

(With two flavors, the condensate leaves a residual SU(2) unbroken gauge symmetry, and not all quarks have a gap. There are still infrared divergences.)

Color-Flavor Locking: Symmetry

Major qualitative features follow from the form of the condensate.

The symmetry ~~breaking~~ transmuting condensate, with flavor and color indices displayed:

$$<\mathbf{q}^{\alpha}{}_{\mathbf{a}}\, \mathbf{q}^{\beta}{}_{\mathbf{b}}> = \kappa_1\, \delta^{\alpha}{}_{\mathbf{a}} \delta^{\beta}{}_{\mathbf{b}} + \kappa_2\, \delta^{\alpha}{}_{\mathbf{b}} \delta^{\beta}{}_{\mathbf{a}}$$

Highly analogous to He^3 B phase.

diagonal symmetry

** Color × Flavor_L × Flavor_R → Flavor'_{L+R} **

$$\langle \mathbf{1}|(q_a^\alpha)_L^i(\vec{k})(q_b^\beta)_L^j(-\vec{k})|\mathbf{1}\rangle =$$
$$\epsilon^{ij}\left(v_1(|\vec{k}|)(\delta_a^\alpha\delta_b^\beta - \delta_b^\alpha\delta_a^\beta) + v_2(|\vec{k}|)(\delta_a^\alpha\delta_b^\beta + \delta_b^\alpha\delta_a^\beta)\right) =$$
$$-(L \leftrightarrow R)$$

$$v_1 >> v_2$$

$$\langle \mathbf{U}|(q_a^\alpha)_L^i(\vec{k})(q_b^\beta)_L^j(-\vec{k})|\mathbf{U}\rangle =$$
$$\epsilon^{ij}\left(v_1(|\vec{k}|)(U_a^\alpha U_b^\beta - U_b^\alpha U_a^\beta) + v_2(|\vec{k}|)(U_a^\alpha U_b^\beta + U_b^\alpha U_a^\beta)\right) =$$
$$-(L \leftrightarrow R)$$

CFL Spectroscopy

There are three kinds of elementary excitations: quark field quanta, gluon field quanta, and collective modes.

Quark field quanta "=" baryon octet

Gluon field quanta "=" vector meson octet

Collective modes = Nambu-Goldstone bosons of chiral symmetry breaking "=" pseudoscalar octet

Since the spectrum and the symmetry of CFL superconductivity nicely matches the expectations for the "nuclear" phase in 3-flavor QCD, we are invited to conjecture that there is no phase transition as the density is increased.

Quark-hadron continuity!

It might seem peculiar a quark can have the same properties as a baryon (three quarks), much less that there's a continuous interpolation ...

... but if space is filled with quark pairs, the difference is negotiable.

What about the quarks' fractional electric charges?

CFL Electromagnetism

The original U(1) of electromagnetism is broken, and so is the original SU(3) of color - but a special combination of the two is unbroken.

This is similar to electroweak physics, where both the weak SU(2) and hypercharge U(1) are broken, but a certain combination is unbroken, to give us ordinary electromagnetism.

$$\gamma \ : e \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}_b^a$$

$$\Gamma \ : g \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}_\beta^\alpha$$

$$\tilde{\gamma} \ = \ \frac{g\gamma + e\Gamma}{\sqrt{g^2 + e^2}}$$

$$\tilde{\gamma}|\mathbf{1}\rangle \ = \ 0$$

According to this emergent photon, all charges are integer multiples of the electron charge!

It is a physical realization of the old Han-Nambu charge and color assignments, with integrally charged quarks.

Material Properties

Superfluid (breaks baryon number)!

Transparent insulator (gap for charged excitations)!

Density jump!? (transition from nuclear matter)

Neutron star phenomenology?

For non-degenerate quarks at lower densities, several alternative ground states have been proposed, including some unusual features:

meson condensates

secondary condensates

crystalline superconductivity (LOFF)

gapless superconductivity (breached

Here, as in many aspects of strong interaction physics, we desperately need better methods of calculation.

Summary

QCD at asymptotically high density can be treated analytically, using the techniques of BCS theory.

Confinement, chiral symmetry breaking, and quark-hadron continuity emerge as straightforward consequences.

To make the theory fruitful, we'll need more information on neutron stars, and - especially - better methods of calculation.