# Whence the Beauty of Mathematics?

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**Abstract**

Some thoughts on the origin of our sense of the beauty (or not!) of mathematics.

Lianne Bynum asks

> Could the perception of math as beautiful be an unconscious recognition/response to our own mathematical structure?

I think that explanation gets close, but that it can and should be taken down to earth, and made more precise. Here I'll sketch some ideas I've had on the subject. (I'll have much more to say about them in a forthcoming book.) These ideas are speculative, but I find their simplicity and straightforwardness compelling. If they are correct, they have some important practical consequences.

Our mental apparatus was evolved to help us to adapt to changing conditions in the world, and to steer events toward our desires[1]. To those ends, abilities that empower us to make successful predictions are enormously important. They let us to react appropriately to changes, and to create sensible plans.

Many prediction-making abilities are low-level and innate. We might say that trees "predict" the arrival of winter and decide to shed their leaves, for example. But in discussing the sense of beauty we are dealing with something that is uniquely human, or nearly so, and we should connect it to issues that are characteristically human.

People can make predictions based on logic and induction from experience. Given the survival (or "fitness") value of that ability, evolution will

[1] Yes, I realize that evolution is about reproductive fitness, and that it is run for the benefit of genes, yada yada. The rough, high-level formulation above is good enough for present purposes.

encourage its emergence. By its nature, however, this ability cannot be encoded complete, as a finished product, in heredity, because it requires engagement with the world. For each individual, the ability to make sophisticated predictions is, in important ways, acquired. It is rooted in information-gathering and honed by practice.

To enable this process, evolution gives us incentives. *We feel positively rewarded, when we make successful predictions.* This conclusion connects both to the sense of beauty and to mathematics, and makes a strong link between them. For on the one hand it is rewarding objects or experiences that we call beautiful; and on the other mathematics is a rich source of successful predictions. We can do a sum in several different orders, and confidently expect to get the same answer, for example. More broadly, we can predict that both we and others will arrive at the same answers to a wide variety of mathematical questions, often from different directions.

Of course the joy in doing sums in different orders, or any routine operation, tends to diminish with familiarity. That is consistent with the idea that we are built to feel especially rewarded for skill *acquisition*. Novelty, as well as success, enters into the value of mathematical experience. That is also true for the sense of beauty.

In a few important cases the connection between the sense of beauty, prediction, and our "mathematical structure" is quite direct. Consider, for a notable example, our sense of harmony between musical tones. As far back as 600 BC Pythagoras discovered important mathematical relationships between the tones we sense as concordant, and the length of the lyre strings that produce them. Today we understand that these relationships amount to the statement that we feel that tones sound good together when the associated air-vibrations – and the nerve-firings they trigger in our ear – follow a simple, reproducible, *easily predictable* pattern in time. On the contrary, discordant tones form beats (in the technical sense bit.ly/1f472s0), and fool our sensory system into making false predictions!

Those who wax mystical about the ineffable beauty of mathematics must face up to the embarrassing fact that there are many people who don't find mathematics beautiful at all, and who in fact fear and hate it. From the present perspective, that fact is easy to comprehend. If the sense of mathematical beauty derives from the rewarding experience of successful prediction, then the reverse experience – failure – will be felt as painful. Where math-lovers perceive harmony and beauty, math-haters perceive meaningless, and therefore ugly, noise. The experience of failure in mathematics is especially sharp, because it is so clear-cut: There is a correct answer, and you either get it, and succeed, or you do not, and fail.

If these ideas are on the right track, they have important practical implications for teaching and learning. They emphasize that it is important to reward success, to minimize failure, and to supply novelty. There is obvious tension between the latter two goals. One must introduce novelty, but in digestible doses, leavened by ample rewards. Recommendations of this kind have been made before, of course, but I think they gain force through their integration into broader explanatory ideas. In any case, I find it rewarding to consider the possibility of a beautiful linkage among beauty, prediction and reward – and I predict it will strengthen and endure.