Survey of Symmetry and Conservation Laws (lecture notes)

The theme of symmetry, or invariance under transformation, dominates our understanding of the microcosmos. The more we unveil how Nature works, the more we reveal Her love of symmetry.

In quantum mechanics there are deep connections among symmetry, conservation laws, and "quantum numbers". The most natural way to understand the different particles is to locate them relative to the symmetries they embody. In many ways, the symmetries are more fundamental and more interesting than the particles as such.

In practice, that approach amounts to defining particles by specifying their quantum numbers. That's what we'll do. I'll be spell out how those connections play out for the fundamental ingredients of particle physics, i.e. the (and other important, even if not-so-basic) particles. This will be quick work once we've got the symmetries.

It is important to recognize that there are many variations on the theme of symmetry. We will be discussing, in addition to (possibly) rigorous symmetries: approximate symmetries, spontaneously broken symmetries, methodological symmetries (gauge invariance is of this kind), asymptotic symmetries, anomalous symmetries (mostly next semester). Many of the sharpest questions in particle physics – the kinds where progress leads to Nobel Prizes – including several that are at the forefront of research today, center around the implementation of symmetry, as we'll see.

Here's a little table indicating the different symmetries I'll be surveying in this lecture, with a few orienting remarks about each. More detailed comments on Poincaré symmetry appear in a separate file.

symmetry	conserved quantities	comments
Space-Time (Poincare) Symmetry	energy, momentum, angular momentum, center of mass velocity; particles: mass and spin; m = 0 special: helicity	uniformity of physical law; needs to be exact for consistent general relativity
CPT		existence and "anti-ness" of antiparticles; honorary Lorentz transformation; presumably exact
CP ($\approx T$) charge parity	discrete	violated only in complex weak processes; relevant to cosmological matter/antimatter asymmetry challenge for SUSY
T ($\approx CP$) time reversal		theoretical violation only; forbids electric dipole moments; *not* thermo-paradoxical
P parity	discrete	valid for strong and electromagnetic processes; "maximal" weak violation
C charge conjugation	discrete	valid for strong and electromagnetic processes; "maximal" weak violation
electromagnetic gauge invariance	electric charge	no longitudinal photons; needs to be exact; quantized charge - why?

symmetry	conserved quantities	comments
B baryon number	baryon number; counting	valid for strong, electromagnetic processes; theoretical, very feeble weak violation (instantons); no violation observed, $\tau_p \geq 10^{33}$ yrs.; relevant to unification, cosmology
I_3 third component of isospin	u - d number; counting	valid for strong, electromagnetic processes; weakly trashed, but in orderly ways
$\vec{I}$ isospin	rotations $u \leftrightarrow d$	approximate for strong interactions; electromagnetically and weakly trashed, but in orderly ways; central to nuclear physics
chiral $SU(2) \times SU(2)$	approximation $m_u, m_d \sim 0$	approximate for strong interactions; spontaneously broken – profound theory, useful for pion physics; electromagnetically and weakly trashed, but in orderly ways

symmetry	conserved quantities	comments
S strangeness	s number; counting	valid for strong, electromagnetic processes; weakly trashed, but in orderly ways
flavor $SU(3)$; "eightfold way"	$u \leftrightarrow d \leftrightarrow s$ rotations	very approximate for strong interactions; mature form is quark model
$\mathcal{C}$ charm	c number	similar to S
$\mathcal{B}$ bottom	b number	similar to S
$\mathcal{T}$ top	t number	similar to S
L_e electron lepton number	e + ν_e number	feebly violated in neutrino oscillations – signature of unification?
L_μ muon lepton number	$\mu + \nu_\mu$ number	feebly violated in neutrino oscillations – signature of unification?
L_τ tau lepton number	$\tau + \nu_\tau$ number	feebly violated in neutrino oscillations – signature of unification?
$L_e + L_\mu + L_\tau$ lepton number		theory prefers violation; no violation yet observed – challenging; Majorana neutrinos?