Superfluidity and Symmetry Breaking

An Unfinished Symphony

The Classics

The simplest model for superfluidity involves a complex scalar field that supports a phase (U(1)) symmetry in its fundamental equations, but not in their stable solutions.

This sort of theory* describes the superfluidity of liquid He^4.

The scalar field creates and destroys helium atoms, and the (spontaneously broken) phase symmetry is associated conservation of He^4 atom number.

* I will use relativistic kinematics ...

$$V(\phi) \;=\; -\frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}(\phi^*\phi)^2$$

$$\begin{aligned} |\langle\phi\rangle| &= v \\ v &= \frac{\mu}{\sqrt{\lambda}} \end{aligned}$$

$$\phi \;\equiv\; (v+\rho)e^{i\theta} \;\equiv\; (v+\rho)e^{i\sigma/v}$$

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial^\mu\phi^*\partial_\mu\phi - V(\phi) \\
&\rightarrow \frac{1}{2}(1+\frac{\rho}{v})^2\,\partial^\mu\sigma\,\partial_\mu\sigma + \frac{1}{2}\partial^\mu\rho\,\partial_\mu\rho - \tilde{V}(\rho) \\
\tilde{V}(\rho) &\equiv -\frac{\mu^4}{4\lambda} + \mu^2\rho^2 + \frac{3\sqrt{\lambda}\mu}{4}\rho^3 + \frac{\lambda}{4}\rho^4
\end{aligned}$$

One has soft modes for gentle space-time motion with the ground state manifold, i.e. $\sigma \to \sigma(x, t)$.

In the context of relativistic quantum field theory, these soft modes represent massless particles (Nambu-Goldstone bosons).

Of course, conservation of He^4 atoms is not really violated (in a finite sample).

The observable consequence of the mathematical "violation", is that transport of this quantum number is especially enhanced. That is a heuristic way to understand superfluidity.

Superfluidity is flow mediated by the soft modes or Nambu-Goldstone bosons.

Classic superconductors are described by a gauged version of the same model.

$$
\begin{aligned}
\mathcal{L}_{\text{kin.}} &\rightarrow -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\nabla^{\mu}\phi^{*}\nabla_{\mu}\phi \\
F_{\mu\nu} &= \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \\
\nabla_{\mu}\phi &= \partial_{\mu}\phi - igA_{\mu}\phi
\end{aligned}
$$

Expanding the covariant derivative, we meet

$$
\frac{1}{2}\,(1+\frac{\rho}{v})^{2}\,(\partial^{\mu}\sigma - gvA^{\mu})\,(\partial_{\mu}\sigma - gvA_{\mu})
$$

$$
\rightarrow \quad \frac{g^{2}v^{2}}{2}\,(1+\frac{\rho}{v})^{2}\tilde{A}^{\mu}\tilde{A}_{\mu}
$$

mass term!

(plus nonlinear interactions)

$$
\tilde{A}_{\mu} \equiv A_{\mu} - \frac{1}{gv}\partial_{\mu}\sigma
$$

no change in $F_{\mu\nu}$!

If we view superconductors *from the inside*, we have the theory of a massive photon (and gapped ($\approx$ "massive") fermions - about which, more momentarily).

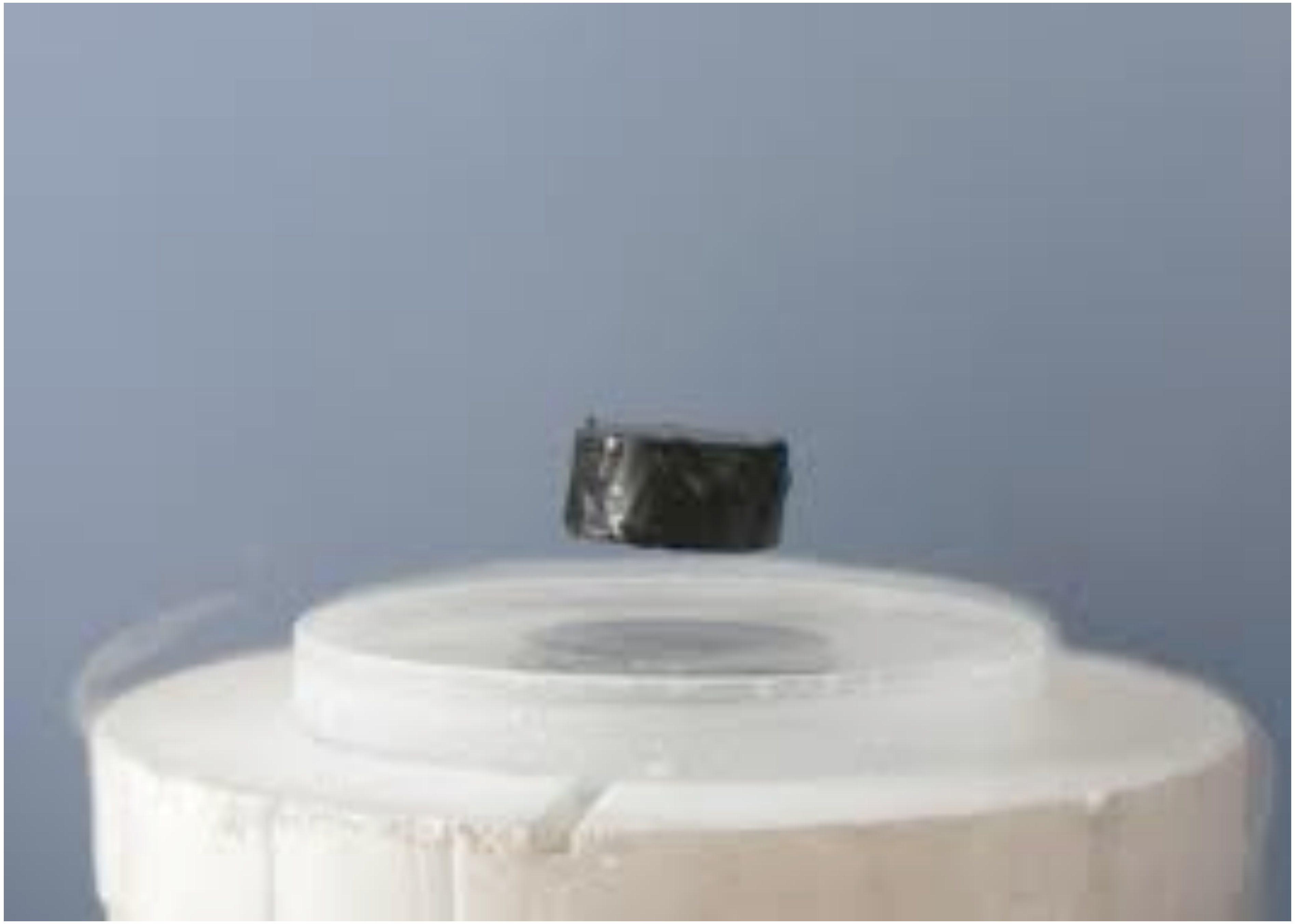
That is the essence of Higgs mechanism of particle physics.

This is not the last time we'll see frontier "particle physics" already embodied in superconductors!

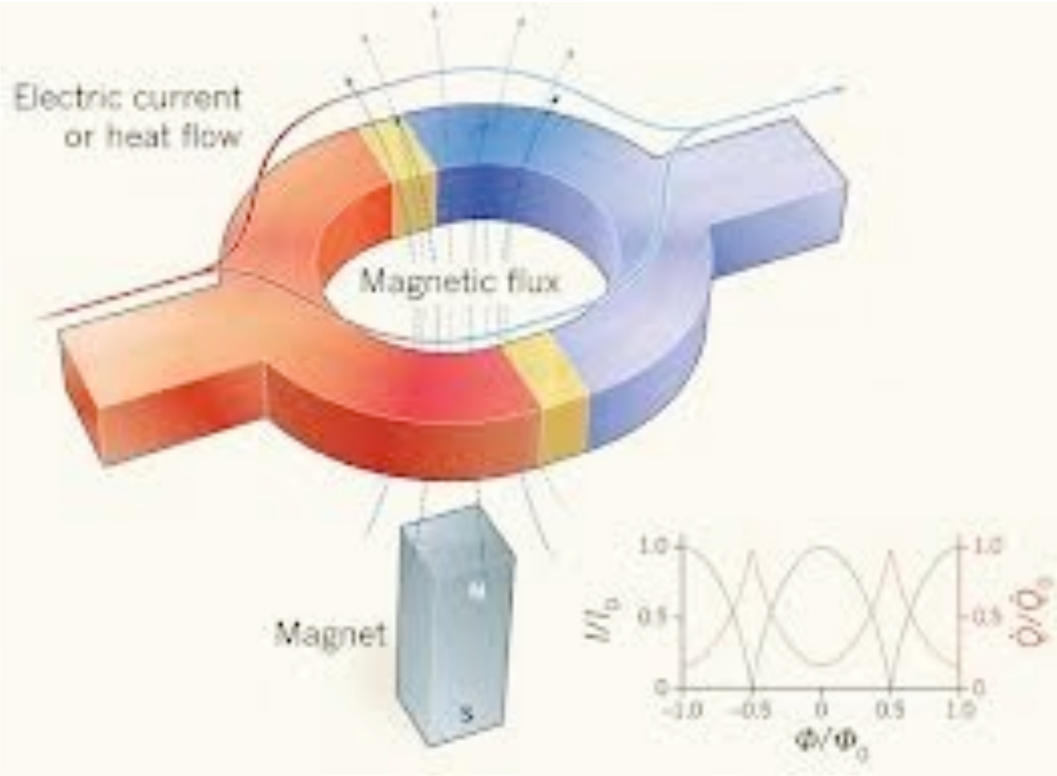
The major bulk features of superconductors - energy gap, massive photon - involve *removing* low-energy dynamics.

If we didn't have independent access to the electromagnetic field, surfaces, and weak links, the superconductor would be featureless.

Fortunately, we do! - Meissner effect, persistent currents and dynamo, Josephson effects ...




CERN
Superconducting


Electric current
or heat flow
Magnetic flux
Magnet
N
S
Φ/Φ0

In simple superconductors, there is no bulk order parameter.

One should speak of "gauge**d** symmetry breaking", *not* "gauge symmetry breaking".

Coming back to gaps and fermion masses:

In the standard model, we see basically

mass + universal coupling

$$y\,\bar\psi\phi\psi \;\stackrel{\sim}{\longrightarrow}\; yv(1+\frac{\rho}{v})\,\bar\psi\psi \;\equiv\; m_\psi(1+\frac{\rho}{v})\,\bar\psi\psi$$

In superconductors, we see basically

$$\begin{aligned}\kappa\,\bar e\bar e e e \quad &\stackrel{\sim}{\longrightarrow} \quad \kappa\langle\bar e\bar e\rangle ee + \mathrm{h.c.}\\ &\equiv \quad \Delta^* ee + \mathrm{h.c.}\end{aligned}$$

mass-like “gap”

The “mass-like” gap is a peculiar, number-violating term.

In fact, it has the same structure as the so-called Majorana mass term, which plausibly, but not surely, governs neutrino masses.

Electrons are Majorana fermions (in superconductors).

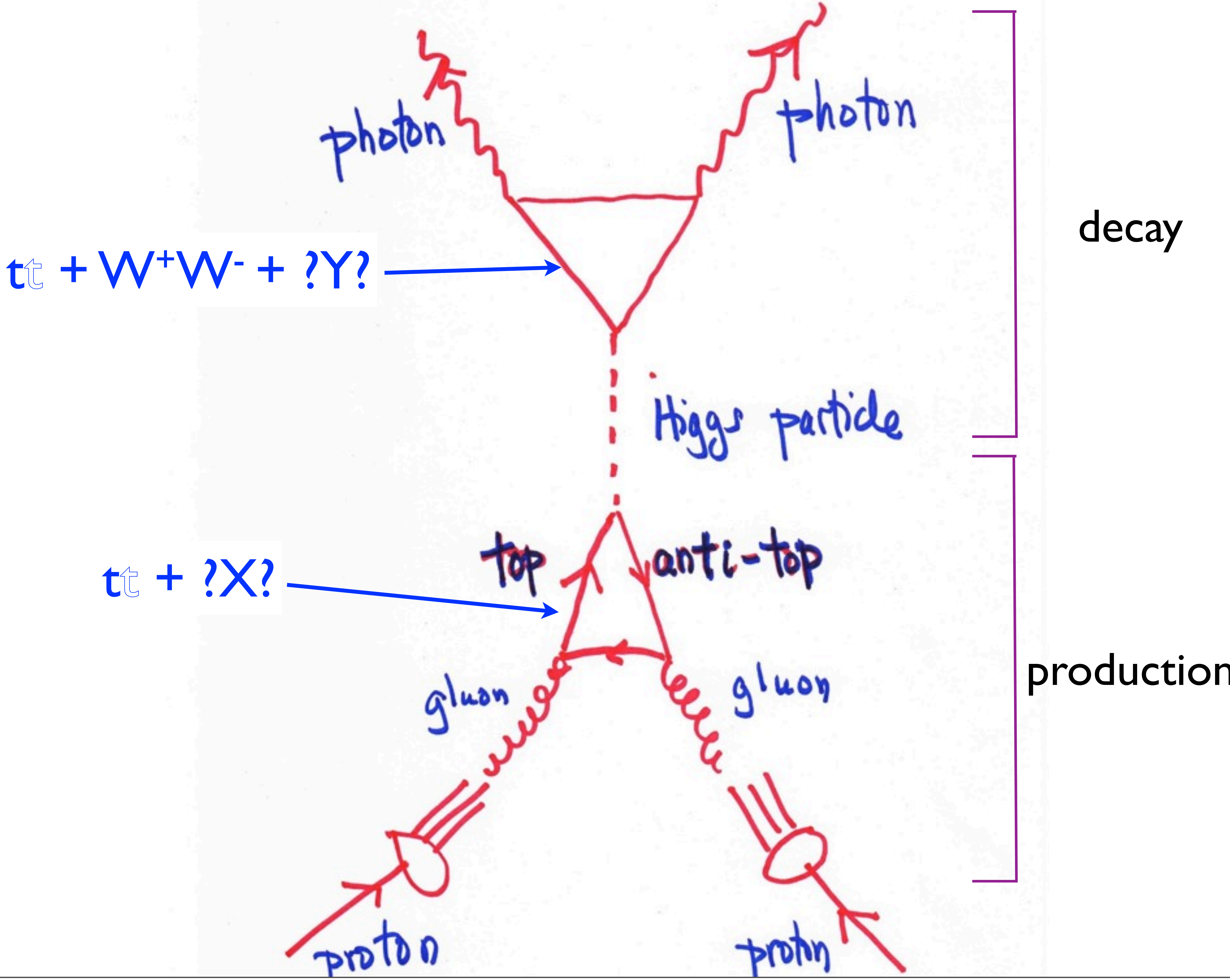
The physical signature of Majorana fermions is that they annihilate in pairs* - unlike, say, neutrons.

Electrons with momenta ±k can dissolve into the Cooper pair condensate!

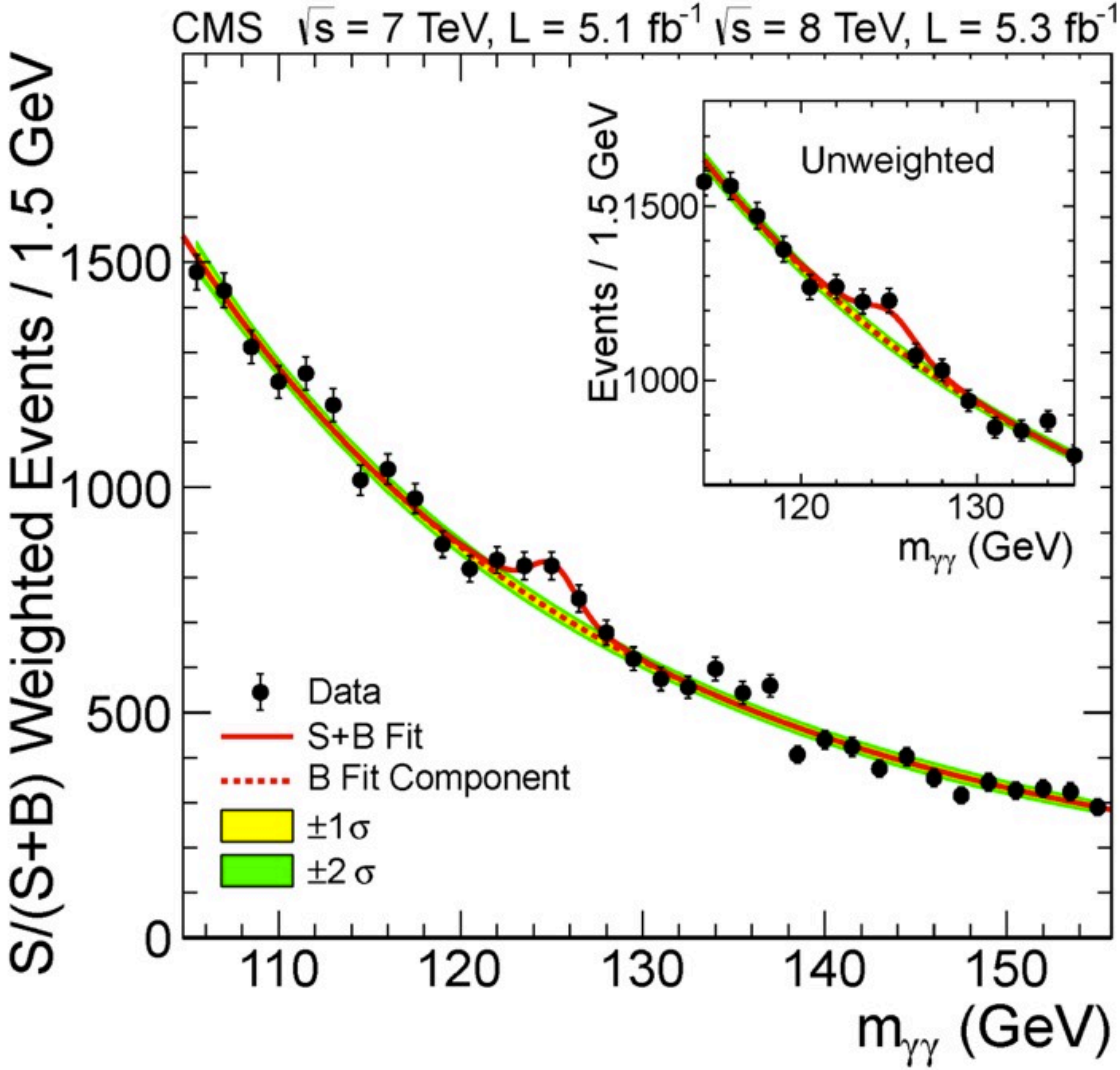
* (More strictly, that pairs have vacuum quantum numbers.)

The ~~Higgs~~ BEH particle, in particle physics, is a lucky by-product of the mass-generating mechanism.

It is the quantum of the magnitude of the condensate (our ρ), which happens to be identifiable.


photon
photon
$t\bar{t}$ + W^+W^- + ?Y?
decay
Higgs particle
top
anti-top
$t\bar{t}$ + ?X?
gluon
gluon
production
proton
proton

The experiments are not easy, but they are convincing.



CMS √s = 7 TeV, L = 5.1 fb⁻¹ √s = 8 TeV, L = 5.3 fb⁻¹
S/(S+B) Weighted Events / 1.5 GeV
Unweighted
Events / 1.5 GeV
Data
S+B Fit
B Fit Component
±1σ
±2 σ
mγγ (GeV)

From Phenomena to Ideas, and Back

The ideas that arose in coming to terms with the classic superfluids are ripe for generalization, and have proved extremely fertile.

By focusing on symmetry and its breaking, we can generate questions and answers in seemingly far-removed fields (as we've already exemplified), and suggest new phenomena.

Moreover Bardeen, Cooper, and Schrieffer (BCS) gave us a simple, powerful microscopic understanding of superconductivity, that is easy to generalize.

(The pseudospin formalism, pioneered by Anderson, provides an especially intuitive and user-friendly approach to BCS theory.)

A very interesting and instructive example, is to analyze the high-density, low-temperature limit of QCD. (Large μ, small T.)

We will see that the central "hard problems" of confinement and chiral symmetry breaking become straightforward in that limit, using superfluidity concepts.

QCD Meets BCS

What is Confinement? What is Superconductivity?

How should we approach high-density QCD?

Let us follow the Jesuit credo: “It is more blessed to ask forgiveness than permission.”

approach from quark-gluon side

apparent simplification - asymptotic freedom

BUT problems (infrared divergences) from massless quarks and gluons

Fortunately, we can stand on the shoulders of giants!

Color Superconductivity

Quark-quark forces can be attractive, so color superconductivity is triggered.

(The totally antisymmetric channel is most favorable.)

Superconductivity-QCD Dictionary

Cooper instability = infrared divergence

Meissner effect = confinement (= Higgs phenomenon)

gap = chiral symmetry breaking

Weak coupling, but nonperturbative =
Nonperturbative, but weak coupling

Since pairing between different flavors can be important, the number of active quark species is relevant, as are quark mass differences.

Three massless flavors give a result that is quite interesting, and clean to interpret.

(With two flavors, the condensate leaves a residual SU(2) unbroken gauge symmetry, and not all quarks have a gap. There are still infrared divergences.)

Color-Flavor Locking: Symmetry

The symmetry breaking condensate, with flavor and color indices displayed:

$$<\mathbf{q}^{\alpha}{}_{\mathbf{a}}\, \mathbf{q}^{\beta}{}_{\mathbf{b}}> = \kappa_1\, \delta^{\alpha}{}_{\mathbf{a}} \delta^{\beta}{}_{\mathbf{b}} + \kappa_2\, \delta^{\alpha}{}_{\mathbf{b}} \delta^{\beta}{}_{\mathbf{a}}$$

$\text{Color} \times \text{Flavor}_L \times \text{Flavor}_R \rightarrow \text{Flavor}'_{L+R}$

Very analogous to He^3 B phase.

$$\langle \mathbf{1}|(q_a^\alpha)_L^i(\vec{k})(q_b^\beta)_L^j(-\vec{k})|\mathbf{1}\rangle =$$
$$\epsilon^{ij}\left(v_1(|\vec{k}|)(\delta_a^\alpha\delta_b^\beta - \delta_b^\alpha\delta_a^\beta) + v_2(|\vec{k}|)(\delta_a^\alpha\delta_b^\beta + \delta_b^\alpha\delta_a^\beta)\right) =$$
$$-(L \leftrightarrow R)$$

$$v_1 >> v_2$$

$$\langle \mathbf{U}|(q_a^\alpha)_L^i(\vec{k})(q_b^\beta)_L^j(-\vec{k})|\mathbf{U}\rangle =$$
$$\epsilon^{ij}\left(v_1(|\vec{k}|)(U_a^\alpha U_b^\beta - U_b^\alpha U_a^\beta) + v_2(|\vec{k}|)(U_a^\alpha U_b^\beta + U_b^\alpha U_a^\beta)\right) =$$
$$-(L \leftrightarrow R)$$

Color-Flavor Locking: Spectroscopy

There are three kinds of elementary excitations: quark field quanta, gluon field quanta, collective modes

Quark field quanta “=” baryon octet

Gluon field quanta “=” vector meson octet

Collective modes = Nambu-Goldstone bosons of chiral symmetry breaking “=” pseudoscalar octet

Since the spectrum and the symmetry of CFL superconductivity nicely matches the expectations for the "nuclear" phase in 3-flavor QCD, we are tempted to conjecture that there is no phase transition as the density is increased.

Quark-hadron continuity!

It might seem odd that a quark can have the same properties as a baryon (three quarks) ...

... but if space is filled with quark pairs, the difference is negotiable.

What about the quarks' fractional electric charges?

Color-Flavor Locking: Electromagnetism

The original U(1) of electromagnetism is broken, and so is the original SU(3) of color - but a special combination of the two is unbroken.

(This is similar to electroweak physics, where both the weak SU(2) and hypercharge U(1) are broken, but a certain combination is unbroken, to give us ordinary electromagnetism.)

$$\gamma \;: e \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}^{a}_{b}$$

$$\Gamma \;: g \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}^{\alpha}_{\beta}$$

$$\tilde{\gamma} \;=\; \frac{g\gamma + e\Gamma}{\sqrt{g^2 + e^2}}$$

$$\tilde{\gamma}|\mathbf{1}\rangle \;=\; 0$$

According to this emergent photon, all charges are integer multiples of the electron charge!

Still Crazy, After All These Years

Two Infant Ideas

Superfluids and Space-Time Translation Symmetry Breaking

In 1964 Larkin, Ovchinnikov, Ferrell and Fulde (LOFF) proposed that possibility of pairing with electrons at a displaced momentum, i.e. pairing k with -k + δ.

In the 21st century, several probable examples have been identified experimentally.

These LOFF states provide sorts of (super-) crystals.

Might one have a similar phenomenon involving non-trivial frequency displacement, yielding *time* crystals?

I think so:

$$
\begin{aligned}
H &= \frac{\varepsilon_2+\varepsilon_1}{2}N + (\varepsilon_2-\varepsilon_1)\,S_3 - g\,(S_-S_+ + S_+S_-) \\
&= \frac{\varepsilon_2+\varepsilon_1}{2}N + (\varepsilon_2-\varepsilon_1)\,S_3 - 2g\,(\vec{S}^2 - S_3^2)
\end{aligned}
$$

$$
N = \sum_k b_k^\dagger b_k + \sum_k a_k^\dagger a_k
$$

$$
\begin{aligned}
S_+ &= \sum_k b_k^\dagger a_k \\
&= S_1 + iS_2 \\
&= S_-^\dagger \\
S_3 &= \frac{1}{2}\Big(\sum_k b_k^\dagger b_k - \sum_k a_k^\dagger a_k\Big)
\end{aligned}
$$

$$\langle \mu, \theta | S_+ | \mu, \theta \rangle \; = \; \Delta_0 e^{i\theta}$$

$$[H, S_+] \; = \; (\varepsilon_2 - \varepsilon_1)\, S_+ \, + \, 2g\,(S_3 S_+ \, + \, S_+ S_3)$$

$$\dot{\theta} \; = \; \varepsilon_2 - \varepsilon_1 + 4g \langle S_3 \rangle$$

Axion Coupling in Superconductors

The axion is a hypothetical particle introduced to explain a striking "coincidence" in the standard model.

It is *almost* a Nambu-Goldstone boson, but its parent symmetry (Peccei-Quinn, or PQ symmetry) has a small intrinsic breaking, through quantum anomalies.

The axion is a BFD*, not least because it plausibly supplies the dark matter in the universe.

* h/t VP Joe Biden

If the right-handed and left-handed electrons have PQ charges b,c then the symmetry current generally will have both vector and axial vector pieces:

$$j_\mu \;=\; (b+c)\,\bar{e}\gamma_\mu e \,+\, (b-c)\,\bar{e}\gamma_5\gamma_\mu e$$

The axion, to a very good approximation, couples linearly to the divergence of this current.

$$\mathcal{L}_{\text{int.}} \;=\; (F+\rho)\, e^{i(b-c)a/F}\, \bar{e}\, \frac{1-\gamma_5}{2} e \,+\, \text{h.c.} \;\xrightarrow{\sim}\; \frac{a}{F}\, \partial^\mu j_\mu$$

The vector current is usually ignored, since its divergence (usually) vanishes. But in a superconductor, it doesn't!

Instead, we get the pretty result

$$\mathcal{L}_{\rm usual} \;=\; i(b-c)\frac{a}{F}m_e\bar{e}\gamma_5 e$$

$$\mathcal{L}_{\rm super} \;\approx\; (b+c)\,\frac{a}{F}\,i(\Delta^* ee - \Delta\bar{e}\bar{e})$$

Heuristically: The axion couples universally to mass, *including superconducting Majorana mass*.

Time-dependent axion fields, as would constitute the cosmic dark matter, can excite pairs over the gap. This might facilitate new detection strategies.

Summary

The connection between superfluidity and symmetry breaking has had a glorious history. It has left us a rich legacy of fertile ideas, that seems far from exhaustion.