Spontaneous Symmetry Breaking: Electroweak

Now let us use the wisdom we've acquired from simpler models to analyze symmetry breaking in the $SU(2) \times U(1)$ gauge theory of electroweak interactions. In this note I will discuss the gauge bosons and their couplings. Fermions, and the family mixing issues they raise, will be discussed separately.

Mixing and Diagonalization

We will concentrate on the quadratic terms in fields, which correspond to momentum, energy, and mass for the field quanta. These terms, when inverted, give the propagators in Feynman rules. Cubic and quartic terms, corresponding to nonlinear interactions, occur as vertices, and determining them presents no difficulties[1].

We have Maxwell-like terms associated with three $SU(2)$ gauge fields, and also with a $U(1)$ hypercharge gauge field. Let us write the potentials of these fields as $B^a_\mu C_\mu$. The Higgs field kinetic term has the form (suppressing isospinor indices)

$$\mathcal{L}_{\text{kin.}} \;=\; \frac{1}{2}(\partial^\mu - igB^{a\mu}\frac{\tau^a}{2} - i\frac{-1}{2}g'C^\mu)\phi^* \, (\partial_\mu + igB^a_\mu\frac{\tau^a}{2} + \frac{-1}{2}ig'C_\mu)\phi \tag{1}$$

reflecting the quantum numbers – weak isospin $\frac{1}{2}$, hypercharge $\frac{-1}{2}$ – of the Higgs doublet. We want to expand ϕ in the form

$$\phi \;=\; \begin{pmatrix} v + \rho + i\sigma \\ \lambda + i\eta \end{pmatrix} \tag{2}$$

where v is the condensate and $\rho, \sigma, \lambda, \eta$ are quantum fields.

Let us focus on the isospinor $(\partial_\mu + igB^a_\mu\frac{\tau^a}{2} + \frac{-1}{2}ig'C_\mu)\phi$. There are no purely numerical contributions. For the contributions linear in the fields, we have from the upper component of this isospinor

$$\partial_\mu\rho + i(\partial_\mu\sigma + \frac{gv}{2}B^3_\mu - \frac{g'v}{2}C_\mu) \tag{3}$$

and from its lower component

$$(\partial_\mu\lambda + \frac{gv}{2}B^2_\mu) + i(\partial_\mu\eta + \frac{gv}{2}B^1_\mu) \tag{4}$$

We must square these out, and interpret the resulting mixture of kinetic (gradient) terms and mass (non-gradient) terms.

[1] There are subtleties associated with fixing a gauge choice, so that the kinetic terms can be inverted, while maintaining a gauge-invariant measure. Careful treatment requires calculation of an appropriate Jacobian, which leads to additional vertices and fields that appear only in loops (the Fadeev-Popov ghosts). These complications are treated in more advanced courses in quantum field theory.

We easily recognize that ρ has a normal, "trivial" kinetic term $\frac{1}{2}\partial^\mu \rho \partial_\mu \rho$. It will give us a neutral scalar field. Recalling our analysis of simpler models, or by fresh calculation, we readily conclude that the ρ is a massive field, with its mass term arising via quadratic terms in the expansion of the potential $V(\phi)$ around its minimum. ρ is, in fact, often celebrated as *the* Higgs field, since its quanta are Higgs particles. From our larger vantage point, it appears as one of four components of a multiplet.

The terms in Eqn. (4) are also of a familiar form, from our treatment of the basic gauged $U(1)$ model. Following that analysis, we understand that the λ and η fields are "eaten" by the vector bosons B^2, B^1, and that after that meal these bosons become massive, with common mass

$$m_W = \frac{gv}{2} \tag{5}$$

In writing this as m_W, I am recognizing that linear combinations $\frac{1}{\sqrt{2}}(B_1 \pm iB_2)$ of these fields, which raise and lower the third component of weak isospin, correspond to the $W^\pm$ bosons coupled to charged weak currents.

Finally, the slightly tricky term is

$$(\partial_\mu \sigma + \frac{gv}{2} B^3_\mu - \frac{g' v}{2} C_\mu)^2 \tag{6}$$

According to our recently acquired wisdom, this will eat the σ field and generate a mass for a vector boson. The new wrinkle is that that the vector boson which acquires mass is a mixture of B^3 and C. That will be the Z boson. A different mixture of B^3 and C quanta will not acquire mass – that mixture will be the photon A.

Since we have defined the Maxwell terms with conventional normalizations, the same for B^3 and C, we are allowed to make orthogonal rotations between these fields, and still have normalized kinetic terms for the new combinations. The rotated fields that diagonalize the mass term we find from Eqn. (6) are evidently

$$\begin{aligned} Z &= \frac{gB^3 - g'C}{\sqrt{g^2 + g'^2}} \\ A &= \frac{g'B^3 + gC}{\sqrt{g^2 + g'^2}} \end{aligned} \tag{7}$$

and we have

$$\begin{aligned} M_Z &= \frac{v\sqrt{g^2 + g'^2}}{2} \\ M_A &= 0 \end{aligned} \tag{8}$$

With these identifications the hard work is done, but it is worthwhile to express the phenomenologically important consequences in a more transparent form. To that task, we now turn.

Couplings and Masses of W, Z

Our theory brings in the three quantities v, g, g'. It outputs observables including M_W, M_Z, e, the form of the neutral current that Z couples to, and the Fermi coupling of the old weak interaction theory.

It is convenient and conventional, in phenomenology, to trade the two coupling constants g, g' for e and an angle, the so-called Weinberg angle. The angle is defined by

$$\sin\theta \equiv \frac{g'}{\sqrt{g^2 + g'^2}} \tag{9}$$

To get insight into the emergent photon, and to identify e, we express the A field coupling as derived from its constituents B^3, coupled to gT_3, and C, coupled to hypercharge Y, as

$$A\frac{gg'}{\sqrt{g^2 + g'^2}}(T_3 + Y) \tag{10}$$

from which we conclude

$$\begin{aligned} e &= \frac{gg'}{\sqrt{g^2 + g'^2}} \\ Q &= T_3 + Y \end{aligned} \tag{11}$$

The coupling of the W boson is governed by

$$g = \frac{e}{\sin\theta} \tag{12}$$

The Fermi constant involves g^2/M_W^2, and is therefore directly proportional to $1/v^2$.

For the coupling of the Z boson we have

$$Z(g^2 T_3 - g'^2 Y)/\sqrt{g^2 + g'^2} = Z\frac{e}{\sin\theta\,\cos\theta}(T_3 - \sin^2\theta\, Q) \tag{13}$$

Finally, we record the simple relation

$$M_W = M_Z \cos\theta \tag{14}$$