Einstein's Proof?

In Einstein's *Autobiographical Notes* he recalls

> ... I remember that an uncle told me the Pythagorean theorem before the holy geometry booklet had come into my hands. After much effort I succeeded in "proving" this theorem on the basis of the similarity of triangles; in doing so it seemed to me "evident" that the relations of the sides of the right-angled triangles would have to be determined by one of the acute angles ...

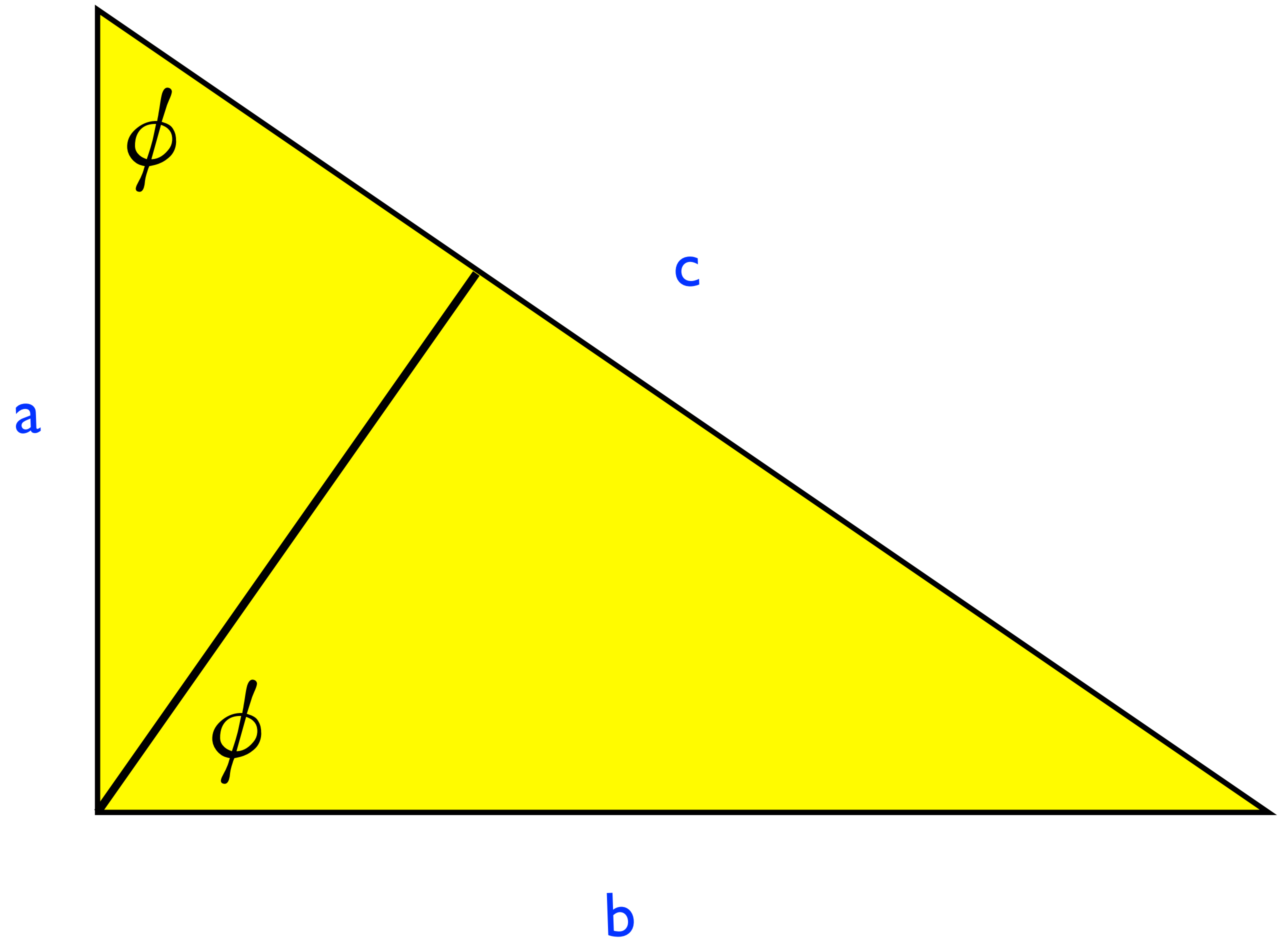
There is not really enough detail in that account to reconstruct Einstein's demonstration with certainty, but "A Polished Jewel" is my best guess. In any case I think it is the simplest and most beautiful proof of Pythagoras' theorem. In particular, this proof makes it brilliantly clear why the *squares* of the lengths are what's involved.

We start from the observation that right triangles that also have a common angle ϕ are all similar to one another, in the precise sense that you can get from any one to any other by an overall re-scaling (magnification or de-magnification). If we re-scale the length by some factor, then we will re-scale the area by the square of that factor.

Now consider the three right triangles that appear in "A Polished Jewel": the total figure, and the two sub-triangles it contains. Each of them contains the angle ϕ, so they are similar. Their areas are therefore proportional to a^2, b^2, c^2, going from smallest to largest. But since the two sub-triangles add up to the total triangle, the corresponding areas must also add up, and therefore

$$a^2 + b^2 \ = \ c^2 \tag{1}$$

– Pythagoras' theorem pops right out!



A Polished Jewel