Particle Survey (lecture notes)

Having surveyed the continuous space-time, discrete P, C, T, and various additive (counting) symmetries, together with electromagnetic gauge invariance, we are ready to survey the building-blocks of the Standard Model.

We will do this, first, by reference to the imaginary world we arrive at by setting the vacuum expectation value of the Higgs field equal to zero. In this imaginary world the properties of the particles, and in particular their behavior under symmetry transformations, are simpler and more transparent. Then we will restore the complicating effect of the space-filling Higgs condensate, to get back to the real world, where the properties appear in a somewhat scrambled form.

To make the table I will need to introduce the gauge transformations $SU(3) \times SU(2) \times U(1)$, which correspond to the color degrees of freedom in quantum chromodynamics (theory of the strong interaction), and the "weak isospin" and hypercharges of electroweak theory, respectively. We'll be discussing the mathematics of gauge invariance in depth later. For now, a few rough notions will be sufficient:

1. The gauge structures are a generalization of the gauge invariance familiar from the Maxwell equations. We can think of electromagnetic gauge invariance as a local form of the symmetry transformation associated with electric charge conservation. The electric charge conservation symmetry transformation involves multiplying charged states (or, of course, the fields that create them) by a phase proportional to their charge. Gauge invariance allows one to choose the phase independently at different space-time points. (Closing the loop: This decouples the dreaded longitudinal photon.) The big step beyond, due to Yang and Mills, is to generalize this "localizing" construction to more complicated, nonabelian symmetries, analogous to isospin ($SU(2)$) or flavor $SU(3)$. To do that we must introduce new vector bosons, analogous to the photon. There is one such vector boson for each generator of the group. Thus for $SU(2)$ we have three photon-like particles, and for $SU(3)$ eight.

2. Weak isospin $SU(2)$ and color $SU(3)$ are entirely different beasts from the isospin $SU(2)$ and flavor $SU(3)$ of nuclear and hadronic physics. So are weak hypercharge and flavor hypercharge (a notion we won't use much if at all). I will denote the weak isospin as $\vec{\tau}$, to distinguish from $\vec{I}$. (In the literature I is usually used for both, relying on context to remove the ambiguity.) Similarly I will denote the gauge hypercharge $U(1)$ by y.

3. Particles can transform according to different representations of the gauge groups, but given the representation, the couplings to the gauge bosons are fixed, up to one overall coupling constant.

4. In our imaginary condensate-free world, the fundamental spin-$\frac{1}{2}$ fermions have mass $m = 0$. Thus the different helicity components can (and do) have different properties,

without violating Poincare symmetry.

5. Electromagnetic charge is a combination of τ_3 and weak hypercharge Y (and photons are combinations of the corresponding gauge bosons). With our conventions

$$Q = \tau_3 + y \tag{1}$$

Particle(s) and Helicity	SU(3)	SU(2)	U(1)
$u_L, d_L\ ;\ -\frac{1}{2}$	**3**	**2**	$\frac{1}{6}$
$u_R\ ;\ +\frac{1}{2}$	**3**	**1**	$\frac{2}{3}$
$d_R\ ;\ +\frac{1}{2}$	**3**	**1**	-$\frac{1}{3}$
$\nu_{eL}, e_L\ ;\ -\frac{1}{2}$	**1**	**2**	-$\frac{1}{2}$
$e_R\ ;\ +\frac{1}{2}$	**1**	**1**	-1
$N_{eR}\ ;\ +\frac{1}{2}$	**1**	**1**	0
$g\ ;\ \pm 1$	**8**	**1**	0
$W^{\pm}, B\ ;\ \pm 1$	**1**	**3**	0
$C\ ;\ \pm 1$	**1**	**1**	0
h; 0	**1**	**2**	$\frac{1}{2}$
graviton; ±2	**1**	**1**	0

We also have two additional families of fermions. We can include them by the substitutions

$$u, d, \nu_e, e, N_e \ \rightarrow\ c, s, \nu_\mu, \mu, N_\mu \ \rightarrow\ t, b, \nu_\tau, \tau, N_\tau \tag{2}$$

Some notes, whose (important!) meaning will become clearer as we go on:

1. Couplings to h connect different families. If we ignore those couplings, we have a gigantic flavor $U(3)^6$ symmetry, allowing separate family rotations among the 6 independent fermionic entities in our table. (Theoretical fine point: some of the U(1) pieces of these symmetries are anomalous.)

2. The particles N are singlets under all the interactions of $SU(3) \times SU(2) \times U(1)$ interactions. Their significance arises in the theory of neutrino masses, especially in the context of unification. You can see, by eye, how they lend a certain quark-lepton parallelism to the table. Unlike for the situation for the other particles in the table, we don't know much about these guys. Strictly speaking, we don't even know there are three of them! (The theory you get by assuming there are two might be fruitful to explore.)

3. When we turn on the Higgs condensate, the entities u_L and u_R, which were two independent helicity states of massless fermions, combine into a spin-$\frac{1}{2}$ massive fermion. Similarly for d_L, d_R, e_L, e_R, ... The situation for neutrinos is more complex, and we'll reserve discussion on that. To get their electric charges, we use Eqn. (1).

4. The complex Higgs doublet h contains four real components, with $Q = 0, 0, \pm 1$. When we turn on the effects of the condensate, there is considerable re-organization in the gauge boson / Higgs sector. One combination of B and C remains massless, and becomes the physical photon. Another combination joins with a $Q = 0$ piece of h to become the massive spin-1 Z boson. The helicity ± 1 components of the massless W join with the helicity 0 h field of the same charge, to make up the massive spin-1 W bosons. We have one spin-0 particle left over, which is the Higgs particle proper.