This brief note introduces the particle oscillation phenomenon through a simple but not trivial idealization of the (hypothetical) phenomenon of neutron-antineutron oscillations. This is something that people have actually been looking for, though most theorists, me included, think it is a long shot. Nevertheless it is especially interesting for us, as it illustrates principles that arise in other important, more complex contexts, and especially sheds considerable light on the concept of "Majorana" fermions.

Neutron-Antineutron Eigenstates and Time

We will consider neutrons (and antineutrons) at rest. Spin will be a passive bystander, so we leave it implicit. We also imagine turning off the weak interaction, so that neutrons and antineutrons are separately stable, though it is possible for neutron-antineutron pairs to annihilate strongly, *e. g.* into proton-antiproton pairs, or multi-pion states.

Let us treat the interactions that violate baryon number B as a perturbation ΔH. Then we have a pure neutron state $|n\rangle$ with $B = 1$ and its CPT conjugate $|\bar{n}\rangle$ with $B = -1$. The $\pm$ CPT eigenstates are respectively

$$\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}(|n\rangle + |\bar{n}\rangle) \\ |-\rangle &= \frac{1}{\sqrt{2}}(|n\rangle - |\bar{n}\rangle) \end{aligned} \tag{1}$$

When we turn on ΔH, these states will become non-degenerate. If we have matrix elements

$$\langle\bar{n}|\,\Delta H\,|n\rangle \;=\; \kappa \tag{2}$$

then

$$m_+ - m_- \;=\; \kappa + \kappa^* \;\equiv\; 2\delta \tag{3}$$

Thus a neutron produced at time $t = 0$ will evolve according to

$$\begin{aligned} |n(t)\rangle &= e^{-iHt}\,|n\rangle \\ &= e^{-iHt}\frac{1}{\sqrt{2}}(|+\rangle \,+\, |-\rangle) \\ &= e^{-im_+t}\frac{1}{\sqrt{2}}(|+\rangle \,+\, e^{-im_-t}|-\rangle) \\ &= e^{-imt}(e^{-i\delta t}|+\rangle \,+\, e^{+i\delta t}|-\rangle) \\ &= e^{-imt}(\cos\delta t\,|n\rangle + i\sin\delta t\,|\bar{n}\rangle) \end{aligned} \tag{4}$$

where of course $m = \frac{1}{2}(m_+ + m_-)$ is the average mass. In words: it oscillates into a neutron-antineutron superposition.

The eigenstates of the static Hamiltonian – the "particles" of definite mass – are $|+\rangle$ and $|-\rangle$. They are their own antiparticles. Thus they are Majorana fermions. The neutron, or for that matter any normal "Dirac" fermion, can be considered as a superposition of two Majorana fermions.

Experimental Probe

If the particles are not isolated, but interact with the external world, the oscillations will be ordinarily be suppressed. This is because $|n\rangle$ and $|\bar{n}\rangle$ have very different strong interactions with a (non-symmetric) medium, such as nuclear matter. Interactions with the medium introduce an effective mass splitting between $|n\rangle$ and $|\bar{n}\rangle$, which drastically reduces their sensitivity to the tiny off-diagonal transitions introduced by ΔH.

Another, more intuitive way to state this is that the medium frequently "measures" the state of our $|n\rangle$, and projects it onto the object that has the appropriate (that is, the observed) strong interaction, so it remains $|n\rangle$.

We can arrange to produce a free neutron. When it is emitted, it starts as $|n\rangle$. Its internal clock starts ticking, and it potentially builds up an $|\bar{n}\rangle$ component. If we keep it isolated for a long time, say with a magnetic bottle, then bring in another, freshly minted neutron, we can see whether they annihilate. If we wait long enough, and test a lot of neutrons, we may discover oscillations.

Of course for real neutrons we cannot turn off the weak interactions, and that limits the time available for oscillations to build up.

Conceptual Remarks

The mathematics of particle oscillations is the mathematics of coupled pendula, where energy can be transferred from oscillations in one to oscillations in another. The same mathematics is central to the description of K meson and neutrino oscillations. There are many additional interesting complications, but the central idea is common.

The subtlety we saw here, between "free" oscillation and "watched" non-oscillation, is also very general. It comes up, for example, in my recent work on spontaneous breaking of time translation symmetry in material systems.