Remarks on Energy in the Many Worlds

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Abstract

This is a short comment on the issue of energy conservation in the sort of branching universe suggested by the many-worlds interpretation of quantum mechanics. I conclude that there is no glaring problem, but that interesting issues of detail might possibly arise at the border of the interface between microscopic and macroscopic behavior.

On several occasions I've been asked how anything like the "multi-worlds" interpretation of quantum mechanics could be consistent with the conservation of energy. On a couple of those occasions I've been not so much asked as told – including once by a Nobel Prize winner – that "multi-worlds" is obviously nuts, because it is *not* consistent with conservation of energy. After all, where is all the extra energy, hidden away in those other ever-branching worlds, coming from?

A formal answer begins with the observation that in quantum mechanics the energy, like all dynamical quantities, is an operator in Hilbert space, and not a substance in the traditional sense, as the question implicitly presupposes. And it is a general consequence of the formalism of quantum mechanics, that if we have a time-independent (time-translation symmetry invariant) dynamics, the energy operator is conserved. Indeed the basic equation for time-evolution of operators in the Heisenberg picture is

$$\frac{d\mathcal{O}}{dt} \ = \ i[H, \mathcal{O}] + \frac{\partial \mathcal{O}}{\partial t} \tag{1}$$

where H is the Hamiltonian (energy) operator. Now if we put $\mathcal{O} \to H$ and suppose that H has no *explicit* t dependence, then we get

$$\frac{dH}{dt} \ = \ i[H, H] + \frac{\partial H}{\partial t} \ = \ 0 + 0 \ = \ 0 \tag{2}$$

But that "answer" is at a high level of abstraction, and does not address the spirit of the question: Where does the extra energy come from? To get something more tangible, let's consider the toy example mentioned in my "Multiverse" draft [1], which I'll now recall.

Consider the wave function $\psi(x_1, x_2, ..., x_N)$ of a system of particles and suppose and suppose that it decomposes into two pieces

$$\begin{aligned} \psi(x_1, x_2, ..., x_N) &= \phi_1(x_1, ..., x_k) f(x_{k+1}, ..., x_N) \\ &+ \phi_2(x_1, ..., x_k) g(x_{k+1}, ..., x_N) \end{aligned} \tag{3}$$

with

$$\begin{aligned} \int \prod_{j=k+1}^{N} dx_j \, f^*(x_{k+1}, ..., x_N) g(x_{k+1}, ..., x_N) &= 0 \\ \int \prod_{j=k+1}^{N} dx_j \, f^*(x_{k+1}, ..., x_N) f(x_{k+1}, ..., x_N) &= 1 \\ \int \prod_{j=k+1}^{N} dx_j \, g^*(x_{k+1}, ..., x_N) g(x_{k+1}, ..., x_N) &= 1 \end{aligned} \tag{4}$$

Then the expectation value of an observable $\mathcal{O}(x_l)$ that depends only on the x_l with $1 \leq l \leq k$ will take the form

$$\begin{aligned} \langle \psi | \mathcal{O}(x_l) | \psi \rangle &= \int \prod_{j=1}^{N} dx_j \, \psi^*(x_1, ..., x_N) \mathcal{O}(x_l) \psi(x_1, ..., x_N) \\ &= \int \prod_{j=1}^{k} dx_j \, \phi_1^*(x_1, ..., x_k) \mathcal{O}(x_l) \phi_1(x_1, ..., x_k) \\ &+ \int \prod_{j=1}^{k} dx_j \, \phi_2^*(x_1, ..., x_k) \mathcal{O}(x_l) \phi_2(x_1, ..., x_k) \end{aligned} \tag{5}$$

Thus there is no communication between the branches of the wave function based on ϕ_1 and ϕ_2. In this precise sense those two branches describe mutually inaccessible (decoherent) worlds, both made of the same materials, and both occupying the same space.

If we apply this discussion to $\mathcal{O} = H$, we might seem, at first glance, to get into the energy conservation problem, since the expectation values for the different "worlds" add. But we need to take into account the *normalization* of the wave functions. Let us define

$$
\begin{aligned}
\int \prod_{j=1}^{k} dx_j \, \phi_1^*(x_1,...,x_k)\phi_1(x_1,...,x_k) &\equiv |\alpha|^2 \\
\int \prod_{j=1}^{k} dx_j \, \phi_2^*(x_1,...,x_k)\phi_2(x_1,...,x_k) &\equiv |\beta|^2
\end{aligned} \tag{6}
$$

Then the *normalized* expectation value of H, which governs experiments, in the multiverse is

$$
\mathcal{E} \;=\; \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \;=\; \frac{\int \prod_{j=1}^{N} dx_j \, \psi^*(x_1,...,x_N)H\psi(x_1,...,x_N)}{|\alpha|^2+|\beta|^2} \tag{7}
$$

while in the effective "universes" based on ϕ_1, ϕ_2 the corresponding quantities are

$$
\begin{aligned}
\mathcal{E}_1 &= \frac{\langle\phi_1|H|\phi_1\rangle}{\langle\phi_1|\phi_1\rangle} = \frac{\int \prod_{j=1}^{k} dx_j \, \phi_1^*(x_1,...,x_k)H\phi_1(x_1,...,x_k)}{|\alpha|^2} \\
\mathcal{E}_2 &= \frac{\langle\phi_2|H|\phi_2\rangle}{\langle\phi_2|\phi_2\rangle} = \frac{\int \prod_{j=1}^{k} dx_j \, \phi_2^*(x_1,...,x_k)H\phi_2(x_1,...,x_k)}{|\beta|^2}
\end{aligned} \tag{8}
$$

Inserting all these into our basic Eqn. (5), we have

$$
\mathcal{E} \;=\; \frac{|\alpha|^2}{|\alpha|^2+|\beta|^2}\mathcal{E}_1 \;+\; \frac{|\beta|^2}{|\alpha|^2+|\beta|^2}\mathcal{E}_2 \tag{9}
$$

Thus the experimentally relevant measure of multiverse energy is not the simple sum of the two universe energies, but rather a weighted average. It is perfectly consistent with Eqn. (9), for example, to have $\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2$. So the answer is that the "extra energy" doesn't have to come from anywhere, because there is no extra energy.

Informally, we may say that if the other universes are inaccessible, they cannot be sources or sinks of energy.

I think that these simple observations take the bite out of the "paradox" of energy conservation in multi-worlds. They hardly do justice, however, to the complexity and interest of the subject. In the discussion above, I assumed that H (as an exemplar of the $\mathcal{O}$ in Eqn. (5)) only depended on a subset of the dynamical variables, and I assumed that the factorization of

ψ was exact and given *a priori*, rather than approximate and dynamically driven. It is possible, I think, that in systems where decoherence is only partial, and changing with time, there might be observable fluctuations that infect the observed time-dependence of energy. But a serious investigation of that question would require work and thought at another level from the preceding.

References

[1] F. Wilczek, "Multiversality", draft available at xx. Soon to appear on the arXiv, and eventually (presumably) in Classical and Quantum Gravity.