# Dimensions Galore!

Recently a mystique has developed around the idea that there might be "extra dimensions" to the physical world. That is indeed a striking concept, but there's no call for mysticism or mystery-mongering around it. Mathematicians and mathematical physicists have used spaces of very large dimension to describe situations in the physical world routinely, and fruitfully, for many decades now. And in the Standard Model, which encodes our best working knowledge of the physical world, extra dimensions are an absolutely central feature, as I will presently explain.

## Entangled

First let me describe how very very high dimensional spaces are at the heart of quantum mechanics.

Some uses of high-dimensional spaces have the appearance of convenient tricks, in that the spaces introduced have little resemblance to traditional ideas about what we commonly mean by space – i.e., roughly speaking, a receptacle through which bodies move. For example, *configuration space* is used to describe the positions of $N$ particles simultaneously, so that a "point" in configuration space describe the state of the whole $N$-particle system. If the positions of particles are specified by 3 coordinates, then the configuration space configuration space describing the positions of all $N$ will be $3N$ dimensional. Configuration spaces were developed extensively in the nineteenth century, as the proper arena for describing the possible physical states of complex systems, allowing identification of regularities among the relative probabilities for different states to occur (statistical mechanics). Configuration spaces take on a new and more vibrant reality in quantum theory, because the wave functions of quantum systems are functions on their configuration space. The probability (or, more properly, the probability amplitude) for two particles $A, B$ to be observed at positions $x_A, x_B$ is not, in general, a product of probabilities for $A$ to be observed at $x_A$ and $B$ to be observed at $x_B$. To describe the correlated probabilities, we need a wave function $\Psi$ that lives on the six-dimensional configuration space

$$\Psi \ = \ \Psi(x_A, x_B) \tag{1}$$

whereas generally we won't be able to write

$$\Psi \ = \ \psi(x_A)\,\psi(x_B) \qquad \text{(Wrong!)} \tag{2}$$

This is the essence of *entanglement*, another wonderful yet very well-established and commonly used aspect of physics – and practical chemistry! – that has developed a mystique recently, after decades of spectacularly successful, routine use.

## Spatial: *What* From *Where*

Now let's return to dimensions of a more conventional sort, the kind accessible to individual particles. To assess this question, we have to ask what additional dimensions "look like". Clearly, it's not a matter of the particle being in different positions, in the everyday sense. Positions, in the everyday sense, are the business of ordinary dimensions – and after all these millennia there's not much room left for crude surprises there! Rather, the key concept is that *where determines what*. As a given "ur-particle" moves in the extra dimensions, other of its properties will change, not its position in the everyday sense. For example, to be specific, a given will might appear to be a red quark, a white quark, or a blue quark, depending on its location within the extra dimensions associated with QCD color space. That color space, in which quarks move, is quite a literal feature of the mathematical formulation of the Standard Model. Electrons, on the contrary, do not have access to color space, which explains why they don't participate in the strong interaction. Positions within other so-called "internal spaces", accessible to some particles but not others, determine other properties of those particles. Electron/neutrinos, and also up/down quarks, for example, are represented by the same ur-particles, when located differently within "weak color" space.

## Superspatial

The basic hypothesis of supersymmetry is similar, but with a twist. In supersymmetry, the extra dimensions are so-called quantum dimensions. The quantum dimensions are the internal dimensions of superspace. The new characteristic of quantum dimensions is that we use numbers with unusual properties, so-called Grassmann coordinates, to describe positions in superspace. (Grassmann coordinates satisfy the multiplication law $xy = -yx$, instead of the familiar $xy = yx$.) As a consequence, an ur-particle can have quite drastically different properties – and appear as different "elementary particles" – depending on its position in superspace. At present, supersymmetry is a hypothetical feature of reality, though there is substantial circumstantial evidence for it (unification of couplings – one of my pride and joys: http://bit.ly/1dT3SsD).

## Folded Up?

There are speculations to the effect that some or all of the internal dimensions of the Standard Model can be described as tiny, folded-up versions of extra dimensions that, before the folding, were on the same mathematical footing as the familiar three spatial dimensions. But whereas those ideas remains highly speculative, the *existence* of additional dimensions in several varieties, as described above, is among the most firmly established results of modern physics.