Grand Normalization Theorem

Over the last several lectures, we have discussed how very general considerations based on principles like locality, relativity, quantum theory, and gauge symmetry, together with the restriction to interactions with mass dimension ≤ 4, together of course with a specified field content, dictate the precise form of the $SU(3) \times SU(2) \times U(1)$ standard model. After symmetry breaking (Higgs field condensation) we arrive at an account of microphysics that makes many predictions, all of them correct so far. We must loosen the rules slightly to accommodate neutrino masses, but here too the general framework takes us quite far, and in good directions.

Although I have not discussed gravity much, it too fits naturally into this framework, with a local symmetry and a restriction to low-dimension invariant interactions yielding general relativity. There is no difficulty in using the quantum version of this theory, and astrophysicists can get on with their work, although the equations do break down in ultra-extreme conditions (in the extremely early universe, and in the deep interior of black holes). Reports of an existential crisis in quantum gravity, the death of quantum field theory ... are, as Mark Twain said up reading reports of his death, highly exaggerated.

Not everything is rosy: We do not find an account of the astronomers' dark matter, and there are many more loose ends and free parameters than we'd like. But the standard model provides a profoundly grounded, amazingly economical, astonishingly successful description of a huge swathe of fundamental physics, and will surely provide a fruitful platform for future explorations.

Now I'd like to crown our work by completing the demonstration that the couplings of the standard model, as used in practice, follow from the general principles. This pleasing result also has important consequences, as I'll discuss in the Scholium below.

We've actually done almost everything required, already. We showed that the gauge kinetic terms are unique, up to overall constants, one for each group, that we identified with the usual gauge coupling constants. We showed how the masses and weak mixing angles emerge as the canonical form of general Higgs-fermion couplings, after appropriate field re-definitions. We showed how to form invariant kinetic terms for the matter fields, using covariant derivatives.

The only thing we haven't justified, is taking those invariant kinetic terms with a diagonal, flavor-independent, unit normalization. Our general principles allow, for example, a non-trivial Z matrix in the kinetic terms for left-handed quarks:

$$L_{\text{kin.}} \;=\; Z^r_s \, \overline{Q_{L}}_r \, (i\nabla^\mu \partial_\mu) Q^s_L \;+\; \text{h.c.} \tag{1}$$

wherein all non-flavor indices have been suppressed. We want to justify $Z^r_s \to \delta^r_s$.

The Last Step (Actually the First): Conventional Kinetics

To begin, note that we can assume Z is hermitean. This is because the first term goes over into itself (with $Z \to Z^\dagger$), up to a total derivative, under hermitean conjugation. Now, as we did earlier in our discussion of mass terms, we can re-define the Q_L by a unitary transformation, so as to diagonalize the new Z. Then we can define re-scaled fields, so that the magnitudes of the diagonal terms are all unity. And then we're done.

We should do these redefinitions first, before the unitary re-definitions we use to diagonalize the quark mass matrices. Fortunately unitary transformations in flavor space leave our unit Z intact, so they do not undo that our first stage in normalization, once it is accomplished.

Scholium

1. We must assume that the eigenvalues of Z are positive. This is connected with positivity of total energy.

2. Let us take stock of parameters. Leaving aside neutrino masses, we have three gauge couplings, six quark masses, three charged lepton masses, and four weak mixing parameters (including one CP violating phase), for a total of 16 real parameters. Neutrinos give us three additional masses and six mixing parameters (including one accessible CP violating phase, and two inaccessible ones). So we have $3+6+3+4 = 16$ or $16 + 3 + 6 = 25$ parameters, in the two cases. That sounds like a lot of parameters, and it is a lot of parameters, but there are three crucial differences between this situation and (say) some cockamamie model of the stock market, that has lots of fudge factors. One is that each of the standard model parameters can be isolated and measured separately, second is that we have a wealth of high-quality, precision data to compare, and third is that we do not get to freely invent fudge factors – the nature of the standard model parameters follows from general principles, as we've discussed. It should also be said that most of the parameters are connected with phenomena involving heavy, unstable particles (masses and mixings of heavy quarks and leptons), so that our effective model for ordinary behavior – the foundation of chemistry, biology, material science, engineering of all sorts, astrophysics, and most of cosmology – involves far fewer parameters.

3. The masses and mixings that we finally get to measure, are highly processed versions of the underlying fundamental couplings, emerging after several rounds of re-definition. It may be that simple hypotheses for the underlying couplings correspond to complicated relations among the observed masses and mixings.

4. The fact that low-dimension operators can be put into this canonical form, implies that the canonical form will also be available after renormalization, in the full quantum field theory. One will not find, for example, violation of spatial parity P creeping into the strong interaction.

5. In this discussion we have ignored the dimension 4 gauge interactions ("θ terms") proportional to

$$\mathrm{Tr}\, F_{\alpha\beta} F_{\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \tag{2}$$

   Their consideration leads to a fascinating, important, and still very much unfinished story, leading to axions and a possible solution of the dark matter problem. But a proper discussion of this is both very complicated and inconclusive, and I won't attempt it here.

6. We can extend this analysis in a systematic way, to consider possible interactions of higher mass dimension. We've already explored one such extension, that gave us neutrino masses. A more general meditation follows, as our next topic.