RETROSPECTIVE

Ken Wilson: A scientific appreciation

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This brief retrospective is an overview and celebration of Kenneth Wilson's profound contributions to science, meant to be accessible to a broad scientific audience. Wilson's lecture on receipt of the 1982 Nobel Prize (1) gives a more technical and detailed description of his central work, including historical perspective and extensive references.

Phase transitions, such as the boiling of water, are a familiar phenomenon of everyday life. J. W. Gibbs (2), J. van der Waals (3), and others, in now-classic work, illuminated many aspects of phase transitions. However, one feature, known as critical behavior, stubbornly eluded theoretical understanding even as it revealed astonishing regularities.

By way of illustration, it will be easiest to discuss a concrete special case. As a ferromagnet is heated, its magnetism diminishes, until at a critical temperature (the Curie temperature) it vanishes altogether. Closer study reveals surprising regularities, first clearly codified by B. Widom (4). Just below the critical temperature, the strength of the remaining magnetic field vanishes as a fractional power, called the "critical exponent," of the temperature difference. Amazingly, the same value of the critical exponent is observed in many different materials with vastly different Curie temperatures, a property we call "universality." Thus, critical exponents are nontrivial numbers, characteristic of macroscopic matter, that emerge "from nothing." Why do they exist, why are they universal, and how can we compute them?

Leo Kadanoff (5) supplied a physical picture that answers the first two of these questions. (Please note, however, that Kadanoff 's discussion is considerably more sophisticated than the following caricature.) We can think of the magnet as having domains, within which the underlying spins share a common alignment. Below the critical temperature a single domain permeates the entire sample, and so there is a nonzero net magnetization. Above the critical temperature no single domain permeates the sample, and the magnetization averages to zero. Near the critical temperature there are domains that nearly permeate the sample but eventually

die. We will have colossal domains, permeated by huge subdomains, themselves permeated by very large subdomains. By considering the differently sized domains as material objects in their own right, we discover a plausible explanation for why the emergent structure is both self-similar and self-determining (i.e., scale invariant and universal). August de Morgan's couplet (6) captures the essence of this picture:

Great fleas have little fleas upon their backs to bite 'em,
And little fleas have lesser fleas, and so *ad infinitum*.

Kadanoff 's picture is insightful and compelling, but does not in itself provide a quantitative tool. Wilson's breakthrough work (7, 8) in 1971 did just that. The mathematical technique is too complicated for meaningful presentation here. It is both possible and appropriate, however, to mention the technique's leading ideas.

We have fluctuations on many different length scales that overlap and affect one another. Because we are most interested in behavior at very large scales, we will be satisfied to find an effective theory that averages or "integrates out" the influence of fluctuations smaller than some cut-off. Wilson was able to formulate equations that describe how the effective theory changes as one changes the cut-off slightly. This concept, that one can derive equivalent effective theories with different cut-offs, is called the "renormalization group." At the critical point, where the dynamics is scale-invariant, the form of the effective theory must be independent of the cut-off. That requirement, formulated as an equation, determines the theory. In collaboration with Michael Fisher, Wilson found a simple way to get approximate solutions. Their paper (9), with the wonderful title "Critical phenomena in 3.99 dimensions," launched an avalanche of extensions and applications.

We also must deal with fluctuations in the quantum-field theories relevant to high-energy physics: they are quantum fluctuations, rather than thermal fluctuations, but they pose related mathematical challenges.
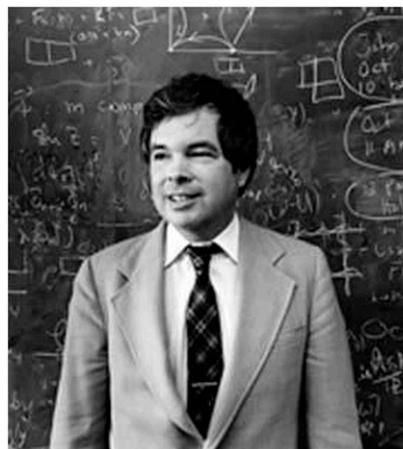

Ken Wilson.

Image courtesy of Division of Rare and Manuscript Collections, Cornell University Library.

In experiments at ever higher energies we probe ever shorter distances. To describe such experiments, we now gradually "integrate in" short-distance fluctuations. In 1969 Wilson sketched (10) how one might apply this version of renormalization group ideas, given a theory of the strong interaction. At Princeton in the spring of 1972, Wilson gave a series of seminars on his recent work, which both David Gross and I attended. Those inspiring lectures informed our subsequent research and helped to lure me from mathematics to physics. Building on Wilson's indications, and working backward, we used observed phenomena to infer the required theory (11). [H. D. Politzer (12) performed the key calculation independently.] That theory, quantum chromodynamics or QCD, is now accepted as the fundamental theory of the strong force.

Wilson was quick to appreciate the promise of QCD, but he realized that to calculate the theory's consequences at (relatively) low energies or long distances would be a demanding task. Wilson's approach (13) was revolutionary in its directness: He formulated the theory in computer-friendly form, essentially as a complicated definite integral in a space of enormously large dimension, and then set out to perform the integral numerically. Many years passed before computers and algorithms were up to the job, but lattice gauge theory amply fulfilled

Wilson's vision. The highest award in the field, awarded annually at the major international conference in Lattice Field Theory, bears Wilson's name.

Wilson loved to work with computers—preferably supercomputers—and was a prophet on their behalf. It is no accident that Paul Ginsparg, the primary architect of arXiv.org, was his student.

In later years Wilson's interests turned toward other challenges. He strove to bring the power of supercomputing and the lessons of critical behavior and QCD to bear on quantum chemistry. In a more radical departure, Wilson proposed to use computers to diagnose, scientifically, the specific difficulties that students encounter in learning, and to overcome them. It would be unwise to bet against Wilson's prophetic visions.

Ken Wilson radiated joy in his work. As I visualize him delivering a seminar, invariably he is smiling.

1 Wilson KG (1993) Nobel Lecture: "The renormalization group and critical phenomena." *Nobel Lectures 1981–1990*, ed Ekspöng G (World Scientific, Singapore). Available at http://nobelprize.org/nobel_prizes/physics/laureates/1982/wilson-lecture.pdf#search='wilson'. Accessed July 15, 2013.

2 Gibbs JW (1874–1878) On the Equilibrium of Heterogeneous Substances. *Transactions of the Connecticut Academy of Arts and Sciences* 3:108–248, 343–524.

3 Van der Waals J (1910) *The Equation of State for Gases and Liquids (Nobel Lecture)*. Available at http://nobelprize.org/nobel_prizes/physics/laureates/1910/waals-lecture.pdf. Accessed July 15, 2012.

4 Widom B (1965) Equation of state in the neighborhood of the critical point J. *Chem Phys* 43:3898–3905.

5 Kadanoff L (1966) Scaling laws for Ising models near Tc. *Physics* 2:263.

6 De Morgan A (1872) A Budget of Paradoxes # 377 (Longmans Green, London).

7 Wilson KG (1971) Renormalization group and critical phenomena. I. Renormalization group and the Kadanoff scaling picture. *Phys Rev B* 4:3174–3183.

8 Wilson KG (1971) Renormalization group and critical phenomena. II. Phase-space cell analysis of critical behavior. *Phys Rev B* 4:3184–3205.

9 Wilson KG, Fisher ME (1972) Critical exponents in 3.99 dimensions. *Phys Rev Lett* 28:240–243.

10 Wilson KG (1969) Non-lagrangian models in current algebra. *Phys Rev* 179:1499–1512.

11 Gross D, Wilczek F (1973) Ultraviolet behavior of non-Abelian gauge theories. *Phys Rev Lett* 30:1343–1346.

12 Politzer HD (1973) Reliable perturbative results for strong interactions? *Phys Rev Lett* 30:1346–1349.

13 Wilson KG (1974) Confinement of quarks. *Phys Rev D Part Fields* 10(8):2445–2459.